A STUDY OF ZONAL AND NODAL PRICES IN POWER SYSTEM MARKETS

by

Wei Liu

A dissertation submitted in partial fulfillment

of the requirements for the degree of

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Abstract

Operation of electric transmission systems within constraints, and relieving equipment overloads when they occur, has become known as "Congestion Management". Power system congestion management is an important issue for Independent System Operators (ISO). Price-based congestion management is widely used in the modern power system markets. Nodal and zonal pricing are the two major pricing systems. Issues of market power and enforcement of price caps also play important roles in congestion management systems.

This thesis examines the unique characteristics of electricity markets, summarizes the similarities and differences between zonal and nodal pricing systems, and describes the zonal congestion management method implemented by the CAISO during 1998-2003.

The thesis summarizes nodal prices calculations, reviews the theoretical fundamentals for nodal prices, and compares different methods to obtain these values. It also illustrates how transmission congestion and losses affect nodal prices.

An OPF model capable of including price caps based on duality theory is developed. This model reveals how price caps affect nodal prices, generation, and load. The concepts are demonstrated using the "New England 16 bus test system."

Finally, an Automatic Zone Creation/Merging/Partition Methodology is proposed. This method exploits the characteristic that the nodal price *patterns* associated with zonal and nodal prices are largely a function of the *network* and do not strongly depend on the *specific*

prices at the various generators. The best organization of zones is not always according to pure geography or company boundaries, but according to electrical connectivity, topology and impedance characteristics of the transmission grid. The method is illustrated by means of case studies based on the California transmission system.

This thesis is dedicated to my dear wife Bo Ren

Acknowledgments

It has been a long journey for me to complete the Ph.D program at the University of Wisconsin – Madison. I cherish the friendships, help, and guidance that accompnied my graduate study.

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Chapter 1

Introduction

1.1 Electricity Market Transmission Congestion

Deregulation of electricity markets is predicated upon competition among suppliers. Competition among suppliers of any commodity requires easy access to customers. In the case of electric power, competition requires that access to the transmission system by generators and loads be managed in a non-discriminatory and equitable manner [1].

To achieve effective competition in electricity markets, two models are of popular interest: the pool model and the bilateral model. The bilateral model is motivated by the concept that free market competition is the best way to achieve competition in an electricity market. This model has also been characterized as one that best achieves the goal of providing customers "direct access" to a supplier of choice. In this model suppliers and customers independently arrange power transactions with each other according to their own financial terms. Economic efficiency is promoted if consumers choose the least expensive generators. This model is a common choice for many commodities other than electricity [1].

The special characteristics of electric power networks introduce two problems that have to be addressed by this model. The first problem relates to the presence of transmission constraints, which strongly suggests that there should exist some form of coordination to maintain system security and make the most efficient use of the constrained transmission system's capacity. The second problem relates to transmission system losses [1]. As a result of these requirements, in a modern power system, the pool model is widely used as the preferred model by Independent System Operator (ISO) to manage open electricity markets.

In California, the system is operated by the California Independent System Operator (CAISO), based on the submission of number of schedules by individuals or individual Security Coordinators (SC). These schedules intended to represent a balance between scheduled demand and scheduled generation. The Preferred Schedule is the Initial Schedule submitted by a SC in the Day-Ahead Market or Hour-Ahead Market. A Preferred Schedule shall be a Balanced Schedule submitted to the ISO by each SC on a daily and/or hourly basis. The individual schedules, however, may lead to possible transmission congestion conditions. Following receipt of a Suggested Adjusted Schedule, a SC may submit to the ISO a Revised Schedule, which shall be a Balanced Schedule, and which shall seek to reduce or eliminate Congestion. There are no Revised Schedules in the Hour-Ahead Market. Transmission Congestion is the condition where there is insufficient transmission capability to simultaneously implement all Preferred or Revised Schedules that Scheduling Coordinators submit to the ISO in the forward markets [9]. Transmission congestion needs to be handled before the open access to the market can be realized. When congestion management is performed, the dynamic security of the system should also be taken into account [2].

Congestion can be relieved, sometimes, by cost free means such as [2]:

- Outage of congested branches (lines or transformers)
- Operation of FACTS devices
- Operation of transformer taps

When it is not possible to relieve congestion by cost-free means, it is necessary to use some non-cost-free means of congestion control methods, which include [2]

- Redispatch of generation
- Curtailment of loads, (demand side management)

If congestion results in either reliability problems, large price differentials or conditions that lead to the exploitation of market power, the long-term solution is to build more transmission lines and more generators. Strategically placed Distributed Generation (DG) can also relieve congestion.

1.2 NERC TLR Procedure

North America Electricity Reliability Council (NERC) proposed the Transmission Loading Relief (TLR) procedures, which is an example of the rule-based congestion management methods, in order to provide a practical method to relieve transmission congestion under the deregulated market [3].

The first step of the procedure is to select the right category of transactions to curtail. The curtailment of progressively higher priority transactions is pursued one category at a time. For example, hourly non-firm transactions are curtailed before daily non-firm transactions, and daily non-firm transactions are curtailed before monthly non-firm transactions. Eventually, even firm transactions are subject to curtailment for purposes of assuring system security and integrity [4].

Next, NERC's TLR uses the weighted impact of each transaction on the overload as the curtailment coefficient, according to the following principle: Within any one category, curtailments are scheduled according to a formula that takes three factors into consideration: the size of the initial consideration; the impact of the transaction on the congested facility; and the amount of relief required. The relative amount of curtailment according to the NERC formula is directly proportional to the total curtailment required, to the size of the initial transaction, and to the Power Transfer Distribution Factor (PTDF) corresponding to the transaction (PTDF is discussed in detail in Section 4.2). That is, a transaction that is most effective in relieving a particular condition is curtailed more than one that has a smaller impact. Likewise, a transaction that is larger is curtailed by a greater amount than a smaller transaction and has the same impact on the constraint. Once all transactions in a given category are exhausted, the algorithm moves to the next curtailment category [4].

The NERC TLR procedure also uses a cutoff factor: Transactions that have less than a 5% impact on the flowgate (flowgate is the line or lines which are likely to be congested) are not considered for curtailment.

In recent years, the TLR recalls have increased dramatically. NERC TLR procedures correctly addressed most engineering concerns pertaining to network security. However, the

TLR rules, which are non-market mechanism, have been widely criticized recently, for not attempting to optimize regional congestion relief [5]. There is also claim that TLR rules have been used in a discriminatory manner.

There are two major concerns about the NERC TLR procedure:

Concern 1: Curtailments are bigger than necessary [4, 5], which invites “gaming”.

Concern 2: Cannot achieve high economic efficiency [5].

NERC stated that the congestion pricing methodology should “ensure that the generators that are dispatched in the presence of transmission constraints must be those that can serve the system loads at least cost, and limited transmission capacity should be used by market participants that value that use most highly” [6]. The most popular solution to this problem is bid-based locational marginal pricing (LMP). The purely administrative TLR procedure cannot satisfy the goal of achieving economic efficiency.

Much research work has been done to improve the NERC TLR procedures [4, 5]. However, the non-market mechanism, such as TLR, cannot replace market mechanism, such as pricing system.

1.3 Price-based Congestion Management Systems

Different pricing systems are used to manage congestion at different electricity markets. The two major pricing systems are nodal pricing and zonal pricing systems.

The basic concept of nodal pricing is simple: the price at a node is the lowest cost way in which power can be delivered incrementally to the node from the present set of marginal units in the system *without violating any of the system constraints in effect.* A nodal price is the market-clearing price at a particular node. Nodal price is also called Locational Based Marginal Price (LBMP) [7], Locational Marginal Price (LMP) [8].

The key reason as to why nodal pricing works well in electricity markets is the fungibility of electricity as a commodity: there is no need to deliver a specific set of electrons from point of production to point of consumption. The transportation of electricity can be replaced with a corresponding amount of electricity at the receiving end (The same principle will not work without modification in networks such as the Internet or for telephone networks. In those cases, the objective is not to deliver a message with general characteristics, but to deliver a *specific* message. Substitution at the receiving end is not possible).

For zonal prices, it is recognized that in meshed networks, zones are only approximations to individual nodal pricing. The objective of creating zones is to closely approximate the "correct" nodal prices under most conditions and where the majority of the "commercially significant" value of the locational prices is captured. In grouping similarly priced nodes into zones, the goal is to simplify producer's and consumer's interactions with the market.

Zones are defined as areas where congestion is infrequent and can be easily priced on an average cost basis. By definition, congestion within zones is infrequent and possibly difficult to predict. Congestion between zones is defined to be frequent with large impacts. Therefore, marginal cost pricing promotes transmission system's efficient use [12].

Zonal pricing begins with a definition of zones. Zone definition is more an art than a precise science, and it is subject to a great deal of judgment. As a minimum, zonal pricing is not unique. It is also subject to assumptions that go beyond the assumptions necessary to create the nodal prices themselves. Zonal pricing requires consideration of both inter-zonal and intra-zonal congestion. A zonal price can be defined in several different approaches, such as non-constrained price followed by uplift, generation based weighted average [9, 13, 14], etc.

The similarities between zonal and nodal pricing far exceed the differences. Both methods are locational pricing systems, and both try to capture economic efficiencies by an appropriate pricing methodology. They begin from the same types of network models and the same general methodologies. But they are different is many ways:

- The zonal model buses are deliberately grouped for the main (claimed) objective of market power reduction, greater liquidity, greater simplicity and more transparency (The fact that zonal pricing leads to less market power, greater transparency and greater liquidity is not universally accepted. But there are valid claims to the contrary).
- The zonal model does not rely on a "central market" that performs all computations for all the markets, where in effect the ISO becomes the market maker as well (It is possible to have nodal pricing without a main market maker making the call on all the prices everywhere. Sounds paradoxical, but it is in fact possible).
- The nodal pricing recognizes different prices at every location, and zonal pricing creates administrative aggregations to reallocate costs. Nodal pricing is preferred for efficiency reasons and to mitigate market power. Zonal aggregation subsidizes the monopolist and increases the profits that can be extracted through the exercise of market power. By

contrast, nodal pricing supports the market and expands the range of tools available to mitigate market power [13].

Debates regarding zonal pricing vs. nodal pricing have been going on for some years [7, 8, 9, 12, 13, 14]. References [13, 14] claim that there are at least four reasons nodal pricing is superior to zonal pricing from a competitive standpoint when the potential for the exercise of locational market power exists:

"*First,* zonal pricing can create market power in the hypothetical zonal dispatch that does not exist in the actual power market under either nodal or inter-zonal pricing. *Second,* zonal pricing can create market power in the zonal redispatch that does not exist in the actual power market under either nodal or inter-zonal pricing. *Third:* by reducing the response of demand in the constrained region to the exercise of locational market power, zonal pricing can make profitable the exercise of market power that would be unprofitable under either nodal or inter-zonal pricing. *Fourt*h, the zonal pricing and redispatch mechanism can reduce the supply elasticity of energy across open increases, making profitable the exercise of market power that would be unprofitable under nodal pricing".

After detailed analysis, it is concluded that "in the choice between market pricing models based on nodal pricing that recognizes different prices at every location, and zonal pricing that creates administrative aggregations to reallocate costs, there is a nearly dominant answer. The result may appear counterintuitive, but nodal pricing is preferred for efficiency reasons and to mitigate market power".

In this thesis, the approaches of defining zonal prices and the inefficiencies of zonal pricing are demonstrated in Chapter 7.

Price caps have been widely used to limit the electricity prices. Improperly set price caps will have negative effects on the system [10]. The details of this issue are discussed in Chapter 6 in this thesis.

1.4 Market Power in Electricity Market

Market power signifies the degree of control that a single firm or a small number of firms have over the price and production decisions in an industry [57, 58]. A more general definition of the market power is given in [62] to account for transmission constraints: "... to reduce profits from production on some units in order to change market prices and profit more from production of other units". To additionally account for the fact that a market participant may have other financial positions whose payouts are tied to spot prices, the following definition in [58] is used:

DEFINITION: A profit-maximizing market participant exercises market power if, for any generator in the market participant's portfolio, its output is shown to be significantly different from that of a profit-maximizing price-taking hypothetical generator with identical cost and operating characteristics at the same location.

A slew of work (a partial list includes [58] – [65]) has been done to study the market power in deregulated markets (especially in California in 2000 – 2001). It is demonstrated that exercising market power could have significant impact in the electricity market. Reference [58] proposes a method to test for the exercise of market power that the authors argue to be practical and accurate. For the purpose of simplicity, this thesis did not consider market power effects during the development of methods, but it need to be considered in the future study.

1.5 Overview of the Thesis

This thesis proposed an Automatic Zone Creation/Merging/Partition Methodology, which is based on the fact that the *nodal price patterns* associated with zonal and nodal prices are largely a function of the network and do not strongly depend on the prices at the various generators. This thesis also developed an Optimal Power Flow (OPF) model including price caps based on the duality theory of Linear Programming. The layout of the thesis is as follows:

Chapter 2 first reviews some basic concepts in microeconomics, and then introduces the unique characteristics of electricity market.

Chapter 3 reviews the congestion management in CAISO. CAISO uses 5% rule to create congestion zones, the details and rationale for this method is discussed. Finally, the price caps in California are studied in detail. All data and background materials used in this chapter are from the CAISO website [9].

Chapter 4 introduces the linear sensitivity analysis method, Power Transfer Distribution Factor (PTDF) and Penalty Factor, and then reviews Optimal Power Flow (OPF) and Kuhn-Tucker Conditions. This chapter covers the most important theoretical fundamentals of calculating nodal prices of electricity.

Chapter 5 introduces two widely used methods of calculating nodal price, the relationship between these two methods are also discussed. An example based on the New England 16 bus system is used to demonstrate the nodal price concepts.

Chapter 6 develops the OPF model including price caps. The duality theory of Linear Programming is used as the vehicle to study the effect of the price caps. The same New England 16 bus system is used to demonstrate the concepts.

Chapter 7 proposes an Automatic Zone Creation/Merging/Partition Methodology. It evaluates the notion of "nodal price patterns", and uses the notion of nodal price pattern to define a methodology for zone creation and partitioning. It is shown that, although nodal prices and zonal prices depend on the cost of generation and many other factors, the *price patterns* associated with zonal and nodal prices are largely a function of the network and do not strongly depend on the prices at the various generators. Thus, it is possible for the most part to separate the concept of zone partitioning from the cost and location of individual generators. This chapter also illustrates some case studies of intra-zonal and inter-zonal congestion zone creation results.

Chapter 8 concludes the thesis and proposes the subjects for future research.

Chapter 2

Characteristics of Electricity Market

2.1 Review of Microeconomics

Before starting the discussion of the electricity market, it is necessary to review some basic theories in microeconomics. Marginal Cost (MC) is the opportunity cost of producing one more unit of good or service. Marginal Revenue (MR) of supplier is the change in total revenue divided by the change in quantity sold [11]. That is, marginal revenue is the change in total revenue resulting from a one-unit increase in the quantity sold.

In perfect competition, when MR = MC, resource use is efficient. Figure 2.1 shows the efficient use of resources. The supply (supplier's cost of production) curve increases with quantity, the demand (customer's value of consumption) curve decreases with quantity. The point of intersection of these curves is where MR = MC. When quantity is Q*, and the price is P*, the resource is used efficiently. When people buy something for less than it is worth to them, they receive a consumer surplus. A consumer surplus is the value of a good minus the price paid for it. When a firm sells something for more than it costs to produce, the firm obtains a producer surplus. A producer surplus is the price of a good minus the opportunity cost of producing it [11]. The consumer surplus and producer surplus are shown in the shaded area in the figure.

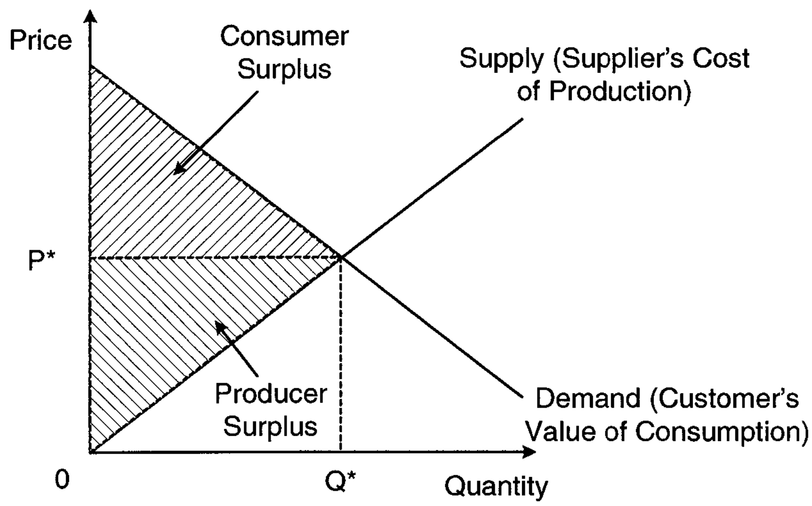

Figure 2.1: Efficient uses of resources

When market power is included, the situation will be much more complicated. In this thesis, only perfect competition is considered.

Figure 2.2 shows the short of supply situation. If the supply is short for some reason, for example, congestion occurs in power system, the quality supplied Q_C is less than Q*. There is deadweight loss. Deadweight loss is the decrease in consumer surplus and producer surplus that results from an inefficient level of production.

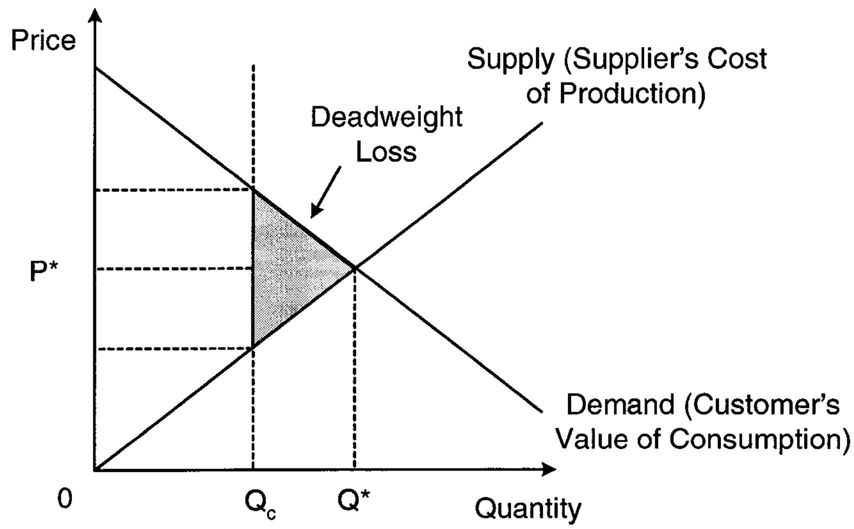

Figure 2.2: The short of supply causes the deadweight loss

Price caps may have significant effects. The inefficiency of the market under inappropriate price cap is shown in Figure 2.3. If price cap is set above the equilibrium price P*, it has no effect. If it is set below P*, a supply shortage ($Q_D - Q_S$) occurs. The opportunity cost of a good includes not only its price but also the value of the search time spent finding the good. People might use resources in search activity equal to the amount they are willing to pay for the available goods, which is area D in the figure [11]. Area D is not the consumer surplus. It represents resources consumed in the searching activity due to the shortage of supply.

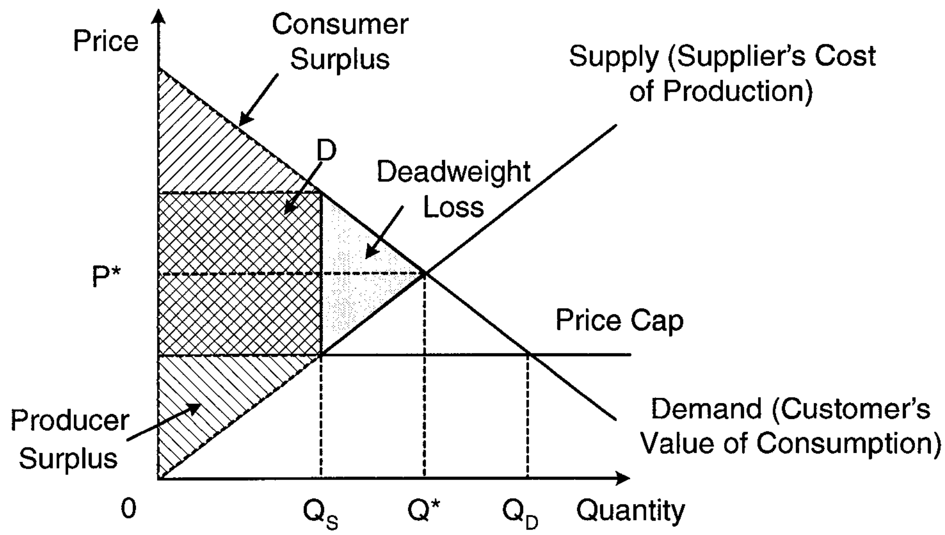

Figure 2.3: The inefficiency of price cap

2.2 Characteristics of the Electricity Power Market

Compared to markets for other goods, the electricity power market is often claimed to be unique [10]. In the short run, the demand is very inelastic, which means that the demand does not change rapidly with change of price. The supply curve is very flat below the supply capacity. When the demand is close to the capacity, the supply becomes very inelastic, which means that the supply can't change much even when price changes a lot. Figure 2.4 shows the supply and demand curves in the typical electricity power market. The supply curve may become vertical at some point. This point is the maximum amount of power the supplier can supply.

Figure 2.4 also shows that very inelastic supply and very inelastic demand intersect at a price that permits producers to cover their capital costs. This price allows the plant to cover its marginal operating costs, earn enough beyond that to justify its capital investments depreciation, and return on investment.

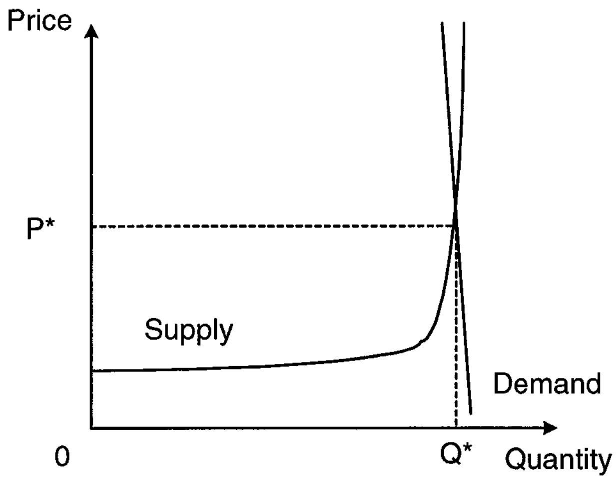

Figure 2.4: The supply and demand curve in the power market

A slight mismatch in which quantity demanded exceeds supply at normal price levels, is shown in Figure 2.5. There are almost no price responsive mechanisms on the supply or demand side that allows the electricity market to adjust to such a mismatch. The price will skyrocket, encouraging a more output as generators run their plants harder – at times risking heavier maintenance costs – due to the tremendous profit opportunity.

Extremely high price can also elicit demand response, but in most current markets, this is precluded or quite limited. The most prevalent sources of "demand responsiveness" are attributable to actions by the Independent System Operator (ISO), which can reduce reserve margins and can exercise interruptible contracts, an extreme measure that causes significant disruption to the affected loads and is thus one that ISOs are reluctant to take.

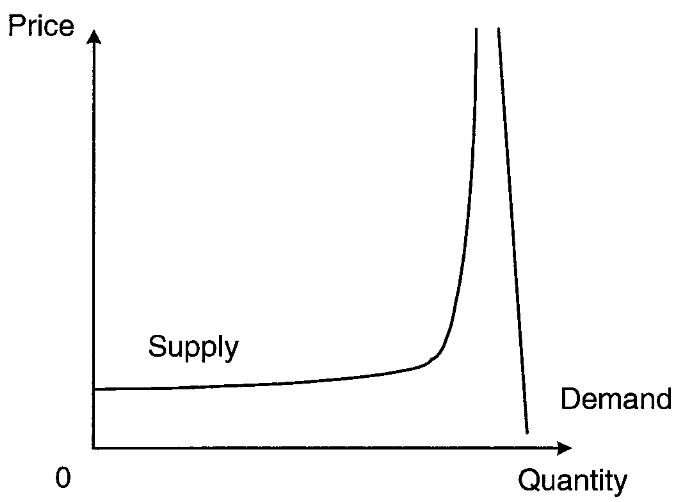

Figure 2.5: A slight mismatch of supply and demand in the power market – quantity demanded exceed supply at normal price levels.

The situation in Figure 2.5 is exacerbated if markets are not completely competitive. Tight supply conditions in electricity markets put sellers in a very strong position to exercise market power, raising price above the level at which competitive supply and demand would otherwise meet. Because market power is easier to exercise in electricity markets when competitive price would have been high, it raises prices more during demand peaks that during off-peak periods. Thus, the presence of market power exacerbates the volatility of prices and further reduces the chance that prices will remain in a reasonable range.

If surplus of capacity exists, the situation is shown in Figure 2.6. There is a mismatch of supply and demand in the opposite direction, with supply exceeding demand at normal prices. With excess supply, price is likely to collapse to the low marginal running costs of the marginal unit. These prices would almost certainly fail to cover the average costs of operating plants. This may result in the shut down of some generating capacity by the suppliers.

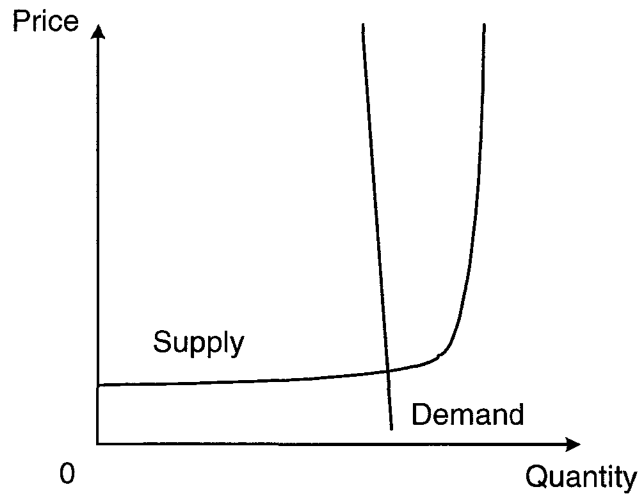

Figure 2.6: A mismatch of supply and demand in the opposite direction, with supply exceeding demand at normal prices.

Chapter 3

Congestion Management In CAISO

3.1 Inter-Zonal and Intra-Zonal Congestion Management

From its inception through the present (early 2004 at the time this document was finalized), the CAISO has managed congestion using a zonal-based approach [15 – 22]. Transmission congestion is divided into two categories:

1. Frequent and costly congestion with widespread effects,
2. Infrequent and inexpensive congestion with localized effects.

The first category is referred to as inter-zonal congestion and primarily occurs on transmission interfaces between congestion zones. The second category is referred to as intra-zonal congestion or AZCM. Therefore, by definition and assumption, the transmission interfaces between zones experience major congestion, whereas congestion zones are network partitions experience minor internal congestion. A congestion zone is a portion of the ISO transmission grid within which transmission congestion is expected to be small and infrequent. Interfaces between zones or Control Area boundaries, on the other hand, consist it makes sense to allocate these congestion costs to customers that utilize the congested paths and thereby send meaningful price signals to such users of the congested interfaces. Subsequent to the creation of the initial congestion zones, the ISO has continuously monitored all congestion management costs to determine if additional zones should be created per the specified zone creation criteria outlined in the ISO tariff.

Transmission Congestion Management is performed sequentially for Inter-Zonal and Intra-Zonal Congestion, using different optimization methodologies and network models. Inter-Zonal Congestion Management is performed first, followed by Intra-Zonal Congestion Management.

The ISO first makes use of the Scheduling Coordinators' (SC)[1] voluntarily submitted incremental and decremental Adjustment Bids in its congestion management protocols to adjust the schedules efficiently. These bids signal to the ISO the value that the SCs place on producing and consuming additional units of energy at different locations on the grid, and when taken together, the value of moving an additional unit of energy between locations.

3.2 CAISO Congestion Zone Definition

The definition of congestion zones is the cornerstone of every zonal congestion model. Poor zone definition may result in excessive loss of market efficiency and poor economic signals that may annihilate the benefits of applying a locational pricing approach to congestion

[1] This thesis describes the situation prevalent in California prior to 2003. Many changes, have taken place since including the effective abolition of Security Coordinators.

management through a zonal model. The CAISO proposed a rigorous process for congestion zone definition that is based on the following principles.

1) Congestion within a zone (Intra-Zonal Congestion) is infrequent and its associated cost over a certain time period does not exceed a specified threshold.

2) Congestion on the transmission interfaces between zones (Inter-Zonal Congestion) is frequent and its associated cost over a certain time period exceeds a specified threshold.

3) Appropriate market power mitigation measures are in place where there is no workable competition within a zone.

4) The locational price dispersion within a zone is small and can be ignored without significant loss of market efficiency.

The congestion cost threshold in the first two principles is currently set at 5% of the transmission interface rating times the corresponding Transmission Access Charge over a year (5% percent rule, which will be discussed in detail later). The third principle, market power mitigation, represents a modification of the CAISO's present tariff provisions, which require that workable competition exist on both sides of a potential inter-zonal pathway in order to create a new active zone. This proposal would relax the workable competition requirement and replace it with a framework for mitigating market power in the absence of a competitive adjustment bid market and competitive Zonal Ancillary Service and Zonal Imbalance Energy markets.

3.3 CAISO Creation of Zones

ISO creates congestion zones in accordance with the ISO Tariff [9, 20]. The following ISO Tariff sections [9] generally describe the process and ISO requirements for establish new zones:

> *7.2.7.2* Modifying Zones. The ISO shall monitor usage of the ISO Controlled Grid to determine whether new Zones should be created, or whether existing Zones should be eliminated, in accordance with the following procedures.
>
> *7.2.7.2.1* If over a 12-month period, the ISO finds that within a zone the cost to alleviate the congestion on a path is equivalent to at least 5 percent of the product of the rated capacity of the path and the weighted average Access Charge of the Participating TOs the ISO may announce its intention to create a new Zone. In making this calculation, the ISO will only consider periods of normal operations. A new zone will become effective 90 days after the ISO Governing Board has determined that a new zone is necessary.
>
> *7.2.7.2.2* The ISO may, at its own discretion, shorten the 12-month and 90-day periods for creating new Zones if the ISO Governing Board determines that the planned addition of new Generation or Load would result in Congestion that would meet the criterion specified in Section 7.2.7.2.1.
>
> *7.2.7.3.5* The determination of whether a new Zone or an existing Inactive Zone should become an Active Zone and the determination of whether a workably-competitive Generation market exists for a substantial portion of the year, shall be made by the ISO Governing Board, using the same approval criteria as are used for the creation or modification of Zones. The ISO Governing Board shall adopt criteria that define a

"workably competitive Generation" market. The ISO Governing Board will review the methodology used for the creation or modification of Zones (including Active Zones and Inactive Zones) on an annual basis and make such changes, as it considers appropriate.

3.4 Active and Inactive Zones

Besides the 5% criterion that was used to form congestion zones, another criterion was further used to determine if a zone would be Active or Inactive. If workable competition is present on both sides of the Inter-Zonal Interface, the zone is active; otherwise it is classified as inactive. Prior to 2003 the ISO had three active congestion zones: NP15, ZP26 and SP15. Both the San Francisco (SF) and Humboldt (HUMB) Zones were declared inactive due to lack of workable competition.

The ISO mitigates congestion on inactive Inter-zonal interfaces by dispatching Reliability Must Run (RMR) units within the inactive zones. RMR units within these inactive zones are typically dispatched to provide incremental energy after final Day Ahead schedules are submitted in order to ensure that sufficient generation within these Inactive Zones is on-line to ensure local reliability. The decremental costs of the units that are being decremented to accommodate the RMR resources are accounted for the imbalance energy market. This approach is similar to performing Intra-Zonal Congestion Management on the inactive Inter-Zonal Interfaces, with two key differences.

First, the decision to dispatch RMR units is based primarily on the need to ensure local system reliability in the event of potential operating contingencies, rather than the need to mitigate Intra-Zonal congestion that may exist each hour. RMR dispatches are also used to mitigate Intra-Zonal congestion over Inactive Inter-zonal interfaces.

Second, RMR costs are not charged to the consumers in the zone through the GOC, but are instead charged to the corresponding PTO. Therefore, these congestion costs are reflected in the PTO access fee paid by all users of the PTO transmission grid.

3.5 Zonal Price Calculation

CAISO first uses a detailed network model to create nodal prices, and then uses this information to calculate zonal prices [18, 22]. It is important to notice that there are significant differences between the California's approaches from the traditional full nodal approach.

First, CAISO enforces a "market separation" constraint that keeps each Scheduling Coordinator's portfolio in balance when re-dispatching resources to resolve inter-zonal congestion. In the traditional problem formulation, there is only a single global power balance constraint that applies to all SCs taken together, instead of being enforced on each SC's portfolio separately.

Second, even though a full nodal network model is used, the branch flow constraints are only enforced for transmission paths between zones. They are not enforced on transmission paths within a zone (actually, these constraints are taken care in the intra-zonal congestion management method).

In a full nodal approach that is not based on the zonal model, all branch flow constraints would be forced simultaneously, possible leading to different price dispersion among the nodes within a zone.

The calculation of the zonal prices involves some form of averaging. The CAISO implemented an approach that calculates a zonal price by using the load-weighted-average.

3.6 The 5% Criterion

3.6.1 Introduction to the 5% Criterion

The 5% criterion is a threshold for the accumulated Intra-Zonal congestion management costs on an Intra-Zonal interface over a period of 12 months. When these costs exceed the threshold, a new congestion zone may be created [20].

The threshold is set to a specified percentage (5%) of the product of the Intra-Zonal interface rating and the weighted-average of the relevant PTO access fees (The weights that are used in the weighted-average are the percentages of ownership of each PTO on the Intra-Zonal interface). This product can be seen as the maximum transmission revenue from the specific Intra-Zonal interface, which would be collected if that interface were fully used throughout the 12-month period. Although the Operating Transfer Capability (OTC) is usually less than the rating of a transmission interface, the rating is used in the criterion because the OTC may vary considerably throughout the year. Therefore, the percentage criterion is the relative portion of the maximum transmission revenue collected from an interface that is considered significant to sacrifice simplicity in favor of market efficiency by promoting the interface to an Inter-Zonal Interface with the creation of a new zone.

3.6.2 Conclusions on Zone Creation Criteria

The basis for the analysis of the zone creation criteria is the comparison of congestion costs to transmission revenue, for a candidate Intra-Zonal interface. This ratio is compared to the ratio of total congestion revenue versus normalized transmission revenue from all existing congested Inter-Zonal Interfaces. The latter ratio or congestion percentage is used as a reference point to assess the results of using the 5% criterion.

Consequently, it is indicated that use of the 5% criterion is reasonable and that the interface should be treated as an active Inter-Zonal interface, if it also meets the second criterion of having workable competition on both sides of the interface.

In the future, ISO will provide more frequent monitoring of congestion cost accumulation and more insight in the determination of appropriate criteria for zone creation. New zones would not be created more frequently than once a year, because of the yearly term of FTRs, but the decisions, and necessary network and system changes for new zones can be made in advance.[2]

Figure 3.1 shows the CAISO zone map, effective from February 2000 [9].

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[2] As of 2003, the CAISO has committed to an eventual switch to a nodal pricing model, thereby obviating the need for many of the issues raised in this dissertation and removing many of the most troubling difficulties and concerns associated with the operation of a zonal-based congestion management system. Other systems, most notably ERCOT in the state of Texas, continue to operate as zonal systems, albeit with a slightly different structure.

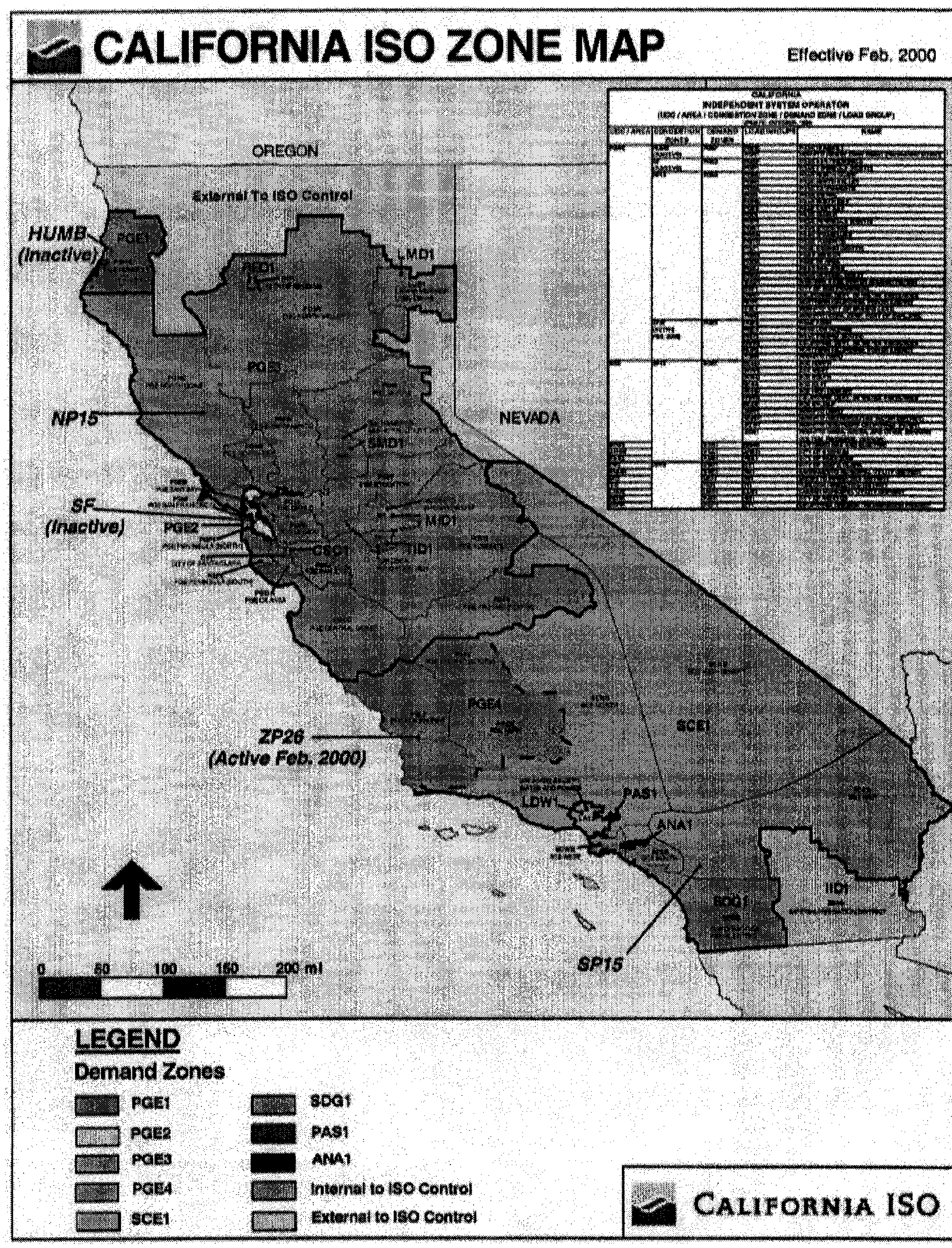

Figure 3.1: CAISO Zone Map [9]

3.7 Price Caps in California Electricity Market

3.7.1 Price Caps in the Electricity Market

In California and other electricity markets, price spikes have motivated policy makers to impose price caps. The debate about the appropriateness of imposing price caps became very intense after summer of 2000 [10].

Price caps are very likely to continue to be a critical element of wholesale electricity markets. Furthermore, the extreme inelasticity of both supply and demand suggest that there is opportunity for exercise of extreme market power, potentially driving prices to orders of magnitude higher than their normal level. Such outcomes would destroy the market. Therefore, the author suggests the debate should be about the level of price caps and mechanisms for their adjustment [10].

Price caps can deter the exercise of market power. Price cap can be set above the competitive price, but below the price that results without cap. This cap will lower prices and increase aggregate output from the firms in the market [23, 24]. As long as the price is set higher than the regional competitive price, firms will have an incentive to provide all the power demanded across the grid without any firm selling at below its production cost. The appropriate level for price caps trades off the risk of setting them too low and deterring production with the risk of setting them too high and permitting the exercise of excessive market power [10].

Those arguing against price caps said that they would reduce investment in production facilities and reduce production from facilities that exist. Both statements are potentially true. If price caps are set too low, they will have detrimental effects. In the short run, a price cap will deter production from an existing facility if the cap is below the short-run marginal cost of production [10].

3.7.2 The History of Price Caps in the California Electricity Market

The price cap in California has been effective from 1998. It has been through four different periods [9]:

- Hard Price Cap (Apr 1, 1998 – Dec 7, 2000)
  - $125/MWh (Apr 1, 1998 – May 26, 1998)
  - $250/MWh (May 26, 1998 – Oct 1, 1999)
  - $750/MWh (Oct 1, 1999 – Jun 30, 2000)
  - $500/MWh (July 1, 2000 – Aug 6, 2000)
  - $250/MWh (Aug 7, 2000 – Dec 7, 2000)
- Soft Price Cap ("Cost-Plus" Price Cap) (Dec 8, 2000 – May 28, 2001)
  - $250/MWh (Dec 8, 2000 – Dec 31, 2000)
  - $150/MWh (Jan1, 2001 – May 28, 2001)
- FERC April 26, 2001 Order (May 29, 2001 – June 20, 2001)
  - Proxy Bids during emergency hours
  - No mitigation in non-emergency hours
- FERC June 19, 2001 Order (June 21, 2001 – Current)
  - Proxy Bid during emergency hours
  - West-wide price limit during non-emergency hours.

3.7.3 Hard Price Cap

Hard price cap was the first one implemented in California. Price caps should be set in a way that takes into account variations in the cost of production. A single rigid price cap that has not included costs of production will either have to be set so high that it has little effect or it will occasionally cause shortages and disruption in the market, as shown in Figure 2.3.

Under the $150 hard cap, because natural gas prices were high in California, many natural gas-fired units would not be able to recover all their operating costs under this proposal and, accordingly, chose to shut down rather than lose money for each kilo-watt-hour sold [25].

In December 2000, FERC approved its order directing remedies for California's wholesale electricity markets. Under the order, sellers bidding at or below a $150 per megawatt-hour breakpoint would receive the market clearing price, but no more than $150 per megawatt-hour. Sellers bidding above this price would be paid their as-bid price but would not be allowed to set the market-clearing price. Furthermore, sellers would be subject to certain reporting and monitoring requirements and, potentially, could be ordered to refund payments that appear to be in excess of their generating costs.

According to microeconomics theory, a plant will continue to operate in the short run as long as it can recover its variable costs. Capital and other fixed costs, such as taxes, will not affect a firm's short-run decisions, because those costs must be paid regardless of whether or not the plant operates - i.e., they are "sunk costs". If a firm cannot recover its variable costs, however, it will stop operating immediately. FERC acknowledged in its December order that costs for natural gas-fired generators would likely be above $150 because of high natural gas prices and NO_x credit prices.

According to the CAISO, generators avoided the capped ISO market in favor of selling into uncapped markets where prices were higher [10, 23]. The California experience demonstrates that if one region has materially lower price caps than a neighboring region, there can be strong short-term incentives for power to flow out of the region with low price caps and some long-term incentives for new generators to be built outside of the low-cap region.

Actually, the price caps in California were frequently violated by ISO, making the California case more complicated. Sometimes, these violations are necessary. For example, in November and December of 2000, the competitive price almost certainly exceeded the cap much of the time. In this case, violation of the cap was the only reasonable action, since it clearly made more sense for most of the generators to shut down than to sell power at $250/MWh [23]. But during the summer, the breaches of the cap by CAISO normally make sellers feel more profitable to exercise market power. When the market was tight and California (and the whole west) is desperate for power, the problem came. If California's price cap were lower than the market-clearing price of the nearby portion of the Western Interconnection, California's generators would sell power to entities outside of California. To resolve the resulting deficiencies for consumers who cannot buy power at prices below the cap, CAISO had to buy out-of-state power at the prevailing market-clearing price, which was usually higher than the price caps. This is called "out-of-market" purchase. This set up an obvious strategy by generators to sell power out of the state and then resell it back in to the state at above the cap. A regional price cap, if it were credible and were set above the competitive price, could be quite valuable in deterring this behavior.

If generators have the ability to strategically withhold supplies in order to drive up prices, the situation can clearly be made worse.

3.7.4 The "Cost-Plus-$25" Price Cap [25]

A second proposal widely advocated is a "cost-plus" price cap, which would allow recovery of each plant's own variable operating costs (fuel, materials, etc.) and provide a payment of $25 per megawatt-hour to all its fixed costs (property taxes, debt payment, profit, etc.). While this proposal is somewhat similar to rate setting under historical cost-of-service regimes, it ignores the relationship between a fixed payment of $25 per megawatt-hour and the amount necessary to recover fixed costs and allow investors the opportunity to earn a reasonable return on capital.

A cost-plus-$25 price cap would have significant effects on new investments. It also could disrupt the operation of existing units unable to maintain the structures of their bond covenants, and could force the abandonment of existing units whose going forward costs (annual fixed operating and maintenance costs, property taxes, etc.) exceed the $25 per megawatt payment.

Although existing units would continue to operate because they would be able to recover their variable operating costs, and because we assume that the $25 per megawatt-hour payment exceeds going forward costs, a firm would not build a new power plant, unless it expected to recover both its variable and fixed costs. Plant developers must consider a number of factors in determining whether or not to build a new plant, including future fuel costs, financing costs, and how quickly the plant can recover its capital costs. In addition, because fixed costs are spread over all the megawatt-hours a plant produces, the number of hours a plant can expect to operate is a critical assumption in estimating profitability.

In conclusion, the "cost-plus-$25" price cap proposal would have a smaller impact immediately but a potentially larger impact over the long term, because it would affect new capacity additions.

3.7.5 The FERC Order in May 2001

In May 2001, FERC issued an order about the California electricity price cap, which was supposed to improve the existing price cap system. The plan sought to ease California's energy crisis. Actually, after-the-fact analysis raises doubts as to whether it was able to ease the electricity shortage or eliminate price gouging. [27]

The major highlights of the plan are:

1. Emergencies only. Once the state has consumed more than 93% of its available power, which is considered as the electricity emergencies, the new cap will go into effect.
2. The cap setting. The amount of cap will vary daily and be based on how much it costs to operate the least efficient plant contributing electricity that day.
3. The plan includes all three types of power emergencies – Stage 1 to Stage 3.

4. The new cap applies only to companies that generate electricity, not to trading firms that buy and resell it at higher prices. So under this rule, the generators create the power with constraint and marketers sell it without constraints.
5. The price cap is not ironclad. A supplier can sell at a higher price as long as it files an explanation with the federal government.
6. The plan requires operators of California power plants to sell their electricity to California "in real time if it is available."
7. The plan requires utilities and state officials who buy power to specify the maximum price they will pay for power and agree not to purchase it for more than that. The idea is to send a message to generators that gouging won't be tolerated.

There are many holes in this order. First, the old cap ($150/MWh) was implemented to power sales at any time, which is more reasonable, since much of the price gouging in the past has occurred at times when power was in ample supply.

Second, this price cap setting policy not only gives power companies little incentive to keep their costs low, but actually encourages them to keep their most expensive, inefficient plants running at all times, to drive up the price of power. So their other low cost, high efficiency plants can make high profits.

Third, the "real time" was interpreted by the energy officials at FERC to be 24 hours up to the point when the electricity is used. But if generators believe the weather forecasts are correct, they can anticipate days when power use will be high and sell their power out of state on that day by hedging. Then on that day, they can say their power is "unavailable" and not subject to this restriction.

Fourth, even the power buyers specifies the highest price they would pay, such promises probably would prove meaningless and could result in that state officials faced with imminent blackouts might just give up and pay whatever power supplies demanded in order to keep the lights on.

In conclusion, inappropriate price caps remove the incentives that the consumers have to be more careful with their consumption of electricity, and remove the incentives for suppliers to add to the supply. The short-term solution could end up with a long-term negative impact.

3.8 Summary

In this chapter, the CAISO zonal management methods in effect in February 2000 are studied. Congestion management is a complicate problem, which involves the generation capacity of generators, transmission capacity, forward market and real-time market, zonal and nodal model, and much more. The Congestion zone definition, Intra-Zonal and Inter-Zonal congestion management, 5% criterion, and price caps are discussed in detail in this chapter.

The overall coordination of the market under proposed zonal management method is not discussed here. Problems such as Long Term Grid Planning, Market Separation, Market Transparency, etc. need further study.

Chapter 4

Linear Sensitivity Analysis and the Optimal Power Flow

4.1 Linear Sensitivity Analysis

Linear sensitivity analysis is widely used in power system analysis. It is a fundamental security analysis tool. In AC power flow, it is valid for small changes in inputs. In DC power flow, it is valid over entire range of the model, but the DC model itself is not valid over the entire range of operation. The method covers two categories of problems. One seeks to predict how the solution will change for variation in the inputs, such as loads, transfers, outages, and voltage support; the corresponding vectors are called tangent vectors. The other seeks to predict how the inputs must change to control the outputs, such as dispatch to control branch flows; the corresponding vectors are called normal vectors [30].

Tangent vectors deal with how does the state change with a change in input, or how does a single action affect all output variables. Examples include Power Transfer

Distribution Factors (PTDF), Flow Distribution Factors, Outage Distribution Factors, and Outage Transfer Distribution Factors.

Normal Vectors deal with how must the input change to maintain a quality of the output state, or how is a single output affected by all combination of inputs. Examples include Loss Factors, Penalty Factors, Meter Multipliers, and Adjustment Factors.

The next two sections discussed the linear sensitivity factors used in this thesis: Power Transfer Distribution Factors (PTDF) and Penalty Factors.

4.2 Power Transfer Distribution Factor (PTDF)

Power Transfer Distribution Factor (PTDF), also called shift factor [33], distribution factor [36], generation shift factor [39], etc., is the sensitivity of a flow to an injection. It reflects the effect of the power injection at an individual bus to the power flow at an individual transmission line, and is useful to find impact of transactions on flowgates.

4.2.1 Calculation of PTDF

The following assumptions are made before performing the calculations:

1. The change of generation at any generator is compensated by an opposite change at the reference bus (a single reference bus is assumed), and that the generation at all other generators remain fixed.
2. Inequality constraints (such as line flow limits, generator capacity) are not considered during PTDF calculation.

3. Both active and reactive power balance constraints are included in the power flow equation set.

Several methods, including exact and approximate, can be used to calculate PTDF [31, 32].

Calculate PTDF from the flow change:

This method is based on the definition of PTDF. The PTDF elements are partial derivatives. This calculation is a finite difference approximation to derivatives. The procedure of this method is:

- Solve power flow, find the base flows of the system
- Change the injection at a particular bus.
- Find new flows
- Divide change of flow by change of injection

The following formula summarizes this method:

$$\mathrm{PTDF}_{ij} = \frac{\text{Power flow change in Line j}}{\text{Power injection at Bus i}} \tag{4.1}$$

Calculate PTDF from Jacobians

This is also an exact method, and is computationally easier. The formula is:

$$\mathbf{PTDF} = \mathbf{J}^{-1}\,(\mathbf{J}^{f})^{\,T} \tag{4.2}$$

where **J** is the ordinary Jacobian of power flow (with respect to bus voltage magnitudes and angles), and $\mathbf{J}^f$ is the Jacobian with respect to line flows. The result is the PTDF matrix, with rows corresponding to the branches, and columns corresponding to the buses.

Calculate PTDF from reactances

This is an approximate method, the formula is:

$$\mathbf{PTDF} = \mathbf{B}^{-1}\,(\mathbf{B}^f)^T \quad (4.3)$$

where **B** is the reduced nodal susceptance matrix and $\mathbf{B}^f$ is the reduced matrix with the branch susceptances. By reduced we mean that rows (and columns) corresponding to a reference location are eliminated (results are insensitive to this choice).

4.2.2. Discussion

PTDF is determined by the system configuration and operating condition. In power system, power balance is required at all time. In most power flow algorithms, when 1 MW is injected at bus j, the system swing bus need to absorb 1 MW. The PTDFs we calculated are all based on this assumption. The value in a PTDF matrix is related to a "reference location", which normally is the system swing bus. If 1 MW is injected and absorbed at the swing bus, power flow at any branch is not affected, so PTDFs at swing buses are zero. In another word, PTDF is a relative value, and the reference location is irrelevant. If we just know where 1MW is injected, but not knowing where it is absorbed, we can't determine how this 1MW injection affects any branch. To get a "bilateral PTDF", the information needed is the difference of an injection and an extraction in a PTDF matrix.

PTDF is a nonlinear variable, using system reactances to calculate PTDF is a linear approximation. The exact PTDF reflects the system information, such as **B** and $\mathbf{B}^f$ (**B** is the reduced nodal susceptance matrix and $\mathbf{B}^f$ is the reduced matrix with the branch susceptances), and it changes with operating point. By using the approximate method, the transient status of the system is not reflected in PTDF, which only changes when system structure changes.

4.3 Penalty Factor

Transmission losses of the system or an area are the difference between total generation and total demand. They play an important role in the economic operation of power systems. Losses change as operating conditions change. They are often assumed to increase quadratically as system loading increases.

The cost of each generator in the power system in servicing load depends upon the location of that generator relative to the loads in the network. Due to transmission losses, a generator that produces power inexpensively but is located a great distance from loads might not be more economical than an expensive generator located close to major loads [36]. Incremental losses lead to penalty factors. Penalty factors disclose the economic efficiency of the transmission network. In this section, the concepts of penalty factor and its calculation are introduced.

Consider a system with n buses. Let buses 1,2, … , m be generation buses, bus 1 be the slack bus. Let λ_i be the incremental cost of operating the generator at bus I [35]. In the absence of network losses, the most economical operating conditions for the network are when:

$$\lambda_1 = \lambda_2 = \dots = \lambda_m \tag{4.4}$$

Let P_L be the network losses and P_{gi} be the power injection at generation node i. If the incremental losses are known, the optimal operating point occurs when:

$$L_1\lambda_1 = L_2\lambda_2 = \dots = L_m\lambda_m \tag{4.5}$$

where L_i is the penalty factor for generator i. These penalty factors can be determined from:

$$L_i = \frac{1}{1 - \frac{dP_L}{dP_{gi}}} \tag{4.6}$$

Define β coefficients:

$$\beta_i = \frac{dP_L}{dP_{gi}} \tag{4.7}$$

A well-established method of calculation of penalty factor in common use by the power utility industry is based on the use of an approximate expression for the line losses in terms of generator bus power, which is called B-coefficients method. The usual expression exploits the fact that for a given operating condition (or base case), the transmission system losses are approximately quadratic in the bus powers at generator buses as follows [35]:

$$P_L \approx \sum_{i=1}^{m} \sum_{j=1}^{m} B_{ij} P_i P_j \tag{4.8}$$

where Bij are constants called the B coefficients or loss coefficients, $B_{ij} = B_{ji}$. From (4.8),

$$\frac{\partial P_L}{\partial P_i} \approx 2 \sum_{j=1}^{m} B_{ij} P_j \tag{4.9}$$

Since by the chain rule of differentiation,

$$\frac{\partial P_L}{\partial P_{gi}} = \frac{\partial P_L}{\partial P_i} \frac{\partial P_i}{\partial P_{gi}} = \frac{\partial P_L}{\partial P_i} \tag{4.10}$$

the penalty factors may be found immediately from (4.10).

Another way to express the simplified B-coefficients method is as follows [38]. Assume the approximate losses as quadratic function of generator injections:

$$P_L = \frac{1}{2} \mathbf{P_g}^T \mathbf{B} \mathbf{P_g} + \mathbf{P_g}^T \mathbf{b} + P_{L0} \tag{4.11}$$

where $\mathbf{P_g}$: m×1, **B**: m×m, **b**: m×1. Calculate the marginal losses:

$$\frac{\partial P_L}{\partial \mathbf{P_g}} = \mathbf{BP_g} + \mathbf{b} \tag{4.12}$$

Penalty factor is:

$$\mathbf{L} = \frac{1}{1 - \frac{dP_L}{d\mathbf{P_g}}} \tag{4.13}$$

Reference [37] provided a thorough survey of the classical methods in economic dispatch. There, the B-coefficients method is critiqued as not accurate, because it does not reflect operating conditions. In many cases, it is not better than neglecting losses. Reference [34] presented transposed Jacobian approach for the direct calculation of penalty factors. It also established a clear relationship between classical B-coefficients methods and the transposed Jacobian methods, which is as shown as follows. Solve power flow:

$$\mathbf{f}(\boldsymbol{\theta},|\mathbf{V}|) = 0 \tag{4.14}$$

Construct Jacobian:

$$\mathbf{J} = \frac{d\mathbf{f}}{d(\boldsymbol{\theta},|\mathbf{V}|)} \tag{4.15}$$

Construct **rhs**:

$$\mathbf{rhs} = -\frac{d\mathbf{P_s}}{d(\boldsymbol{\theta},|\mathbf{V}|)} \tag{4.16}$$

Solve $\boldsymbol{\beta} = \mathbf{J}^T \setminus \mathbf{rhs}^T$, penalty factor is

$$\mathbf{L} = \frac{1}{\boldsymbol{\beta}} \tag{4.17}$$

Reference [36] generalized the concept of penalty factors. It proposed a method to find the row vectors orthogonal to the equilibrium surface in PQ parameter space, which included penalty factors. By appropriate choice of the system parameter vectors in the system equilibrium equations, the penalty factor or β (reciprocal penalty factor) corresponding to real or reactive power injection at any bus can be computed. This method is as follows.

Let $\mathbf{F}$ represent the system equilibrium equations with state variables $\mathbf{z} = (\mathbf{x}, s)$, $\mathbf{x}$ includes bus voltages and angles and generator reactive power outputs but not generation real power outputs; the scalar s represents the system losses or slack generation and can be distributed in any manner so that $\mathbf{F_z}$ is non-singular. $\boldsymbol{\lambda}$ can be any vector of system parameters.

Assume that $(\mathbf{x_0}, s_0, \boldsymbol{\lambda_0})$ is the present stable operating point satisfying the equilibrium equations $\mathbf{F}(\mathbf{x}, s, \boldsymbol{\lambda}) = \mathbf{0}$ and assume that $\mathbf{F_z}|(\mathbf{x_0}, s_0, \boldsymbol{\lambda_0})$ is nonsingular. Then matrix $\mathbf{F_x}$ is a full rank matrix with one more row than column and there exists a row vector $\boldsymbol{\beta}$ such that

$$\boldsymbol{\beta}\, \mathbf{F_x}|(\mathbf{z_0}, \boldsymbol{\lambda_0}) = \mathbf{0} \tag{4.18}$$

$\boldsymbol{\beta}$ is unique up to a scalar multiple. The sensitivity of the losses to a change in the parameter vector in the direction $\boldsymbol{k}$ is found to be:

$$S_\lambda = -\frac{\beta\, F_s|(z_0, \lambda_0)}{\beta\, F_\lambda|(z_0, \lambda_0)\, k} \tag{4.19}$$

The components of **β** are the "betas" or reciprocals of the penalty factors. (4.19) is applied to get penalty factors **L**. This method of calculating penalty factors is used in this thesis.

Another interpretation is that **β** vector determines the normal vector in parameter space to the space to the surface on which the parameters must change to maintain an equilibrium solution. Detailed discussion about this can be found in [36].

4.4 Introduction of Optimal Power Flow (OPF)

Optimal Power Flow (OPF) has a long history in its development [39]. It took a long time and a lot of effort to become a successful algorithm, note that it was first discussed in 1962 [40]. Current interest of these OPF programs focuses on finding the optimal solution based on an objective function, with consideration of security and other constraints of the system.

There are many forms of OPF objective functions. The following are some examples:

- Minimization of the generation cost
- Maximization of the load benefit
- Minimization of the electrical losses in the transmission system
- Minimum shift of generation and other controls from an optimum operating point.
- Minimum load shedding schedule under emergency conditions.

Many constraints can be included in OPF, such as:

- Generator power limits: $LB_{Pi} \le P_i \le UB_{Pi}$, where LB_{Pi} is the lower boundary of P_i, UB_{Pi} is the upper boundary of P_i.
- Generator reactive power limits: $LB_{Qi} \le Q_i \le UB_{Qi}$, where LB_{Qi} is the lower boundary of Q_i, UB_{Qi} is the upper boundary of Q_i.
- Generation and load bus voltage magnitude limits: $LB_{|Ei|} \le |E_i| \le UB_{|Ei|}$, where $LB_{|Ei|}$ is the lower boundary of $|E_i|$, $UB_{|Ei|}$ is the upper boundary of $|E_i|$.
- Flow limits on transmission lines and transformers: $LB_{MVAij} \le MVA_{ij} \le UB_{MVAij}$, where LB_{MVAij} is the lower boundary of power flow from bus i to bus j, UB_{MVAij} is the upper boundary of power flow from bus i to bus j.
- Constraints that represent operation of the system after contingency outages. This special type of OPF is called a "Security Constrained OPF", or SCOPF.

Many adjustable or "control" variables can be used to achieve the objective functions, a partial list of such variables include:

- Generator MW outputs
- Generator Voltages
- LTC transformer tap position
- Phase shift transformer tap position
- Switched capacitor setting
- Reactive injection for a static VAR compensator
- Load shedding
- DC line flow

The ability to achieve different objective functions by adjusting many control variables makes OPF a very important, flexible analytical tool.

The Optimal Power Flow is a very large and difficult mathematical programming problem. Many approaches have been tried in the last several decades. The major methods include:

- Lambda iteration method
- Gradient method
- Newton's method
- Linear Programming method (LPOPF)
- Interior point method.

It is not the intention of this thesis to discuss the details of each method. Interested readers can refer to [39]. Linear Programming is one of the fully developed methods now in common use [39, 41, 42]. Many software packages have been developed to implement this method. It easily handles both equality and inequality constraints. Nonlinear objective functions and constraints are handled by linearization. LPOPF is used in all the work in this thesis.

4.5 Kuhn-Tucker Conditions

The fundamental rule that tells when the optimization is reached in the Linear Programming method is called Kuhn-Tucker conditions, which was introduced in the famous paper by Kuhn and Tucker [41, 39]. The basic of this rule is presented here [39].

Minimize: $f(\mathbf{x})$

Subject to: $\omega_i(\mathbf{x}) = 0, \quad i = 1, 2, \ldots, N\omega$

$g_j(\mathbf{x}) \leq 0, \quad j = 1, 2, \ldots, Ng$

$\mathbf{x}$ = vector of real numbers, dimension = N (4.20)

Then, forming the Lagrange function,

$$L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f(\mathbf{x}) + \sum_{i=1}^{N\omega} \lambda_i \, \omega_i(\mathbf{x}) + \sum_{j=1}^{Ng} \mu_j \, g_j(\mathbf{x}) \qquad (4.21)$$

The conditions for an optimum for the point $\mathbf{x}^0, \boldsymbol{\lambda}^0, \boldsymbol{\mu}^0$ are

1. $\frac{\partial L}{\partial x_i}(\mathbf{x}^0, \boldsymbol{\lambda}^0, \boldsymbol{\mu}^0) = 0,$ for $i = 1, 2, \ldots, N$

2. $\omega_i(\mathbf{x}) = 0,$ for $i = 1, 2, \ldots, N\omega$

3. $g_j(\mathbf{x}) \leq 0,$ for $j = 1, 2, \ldots, Ng$

4. $\mu_j^0 \, g_j(\mathbf{x}^0) = 0$ & $\mu_j^0 \geq 0,$ for $j = 1, 2, \ldots, Ng$

The last condition, often referred to as the complimentary slackness condition, provides a concise mathematical way to handle the problem of binding and nonbinding constraints. Since $\mu_j^0 \, g_j(\mathbf{x}^0)$ equals to zero. Either μ_j^0 or $g_j(\mathbf{x}^0)$ equals to zero, or both equal to zero. If μ_j^0 equals to zero, $g_j(\mathbf{x}^0)$ is free to be nonbinding; if $\mu_j^0 > 0$, then $g_j(\mathbf{x}^0)$ must be zero.

Thus, by looking at μ_j^0, one can get an indication of whether the constraint is binding or not.

In the OPF discussed in the following chapters, Kuhn – Tucker conditions will be used as the fundamental rule to tell if the transmission lines have met the constraints, or if the generators have hit the lower or upper limits.

Chapter 5

Nodal Price of Electricity

5.1 Introduction

Nodal price, also called Locational Marginal Price (LMP), Locational Based Marginal Price (LBMP), is the market-clearing price of electricity at a node. It represents the cheapest way to deliver power to the node under the specific conditions at the time while respecting all limits in effect [44]. The factors affecting nodal price include energy demand, available dispatchable units, economic dispatch, transmission network configuration, transmission constraints, etc. [43]. Reference [45] summarized the earlier work that introduced the concept of spot pricing of electricity, [46] discussed the electric power transmission contracts, and [47] proposed a market mechanism for electric power transmission.

Nodal prices can be obtained as a by-product of an optimization program that seeks to minimize total cost subject to the network constraints, secure operation is ensured by including the appropriate constraints in the optimization. The optimization program can be Optimal Power Flow or Constrained Economic Dispatch. Traditional use of these programs put emphasis on generation production, but actually, the implicit price is just another aspect of the same problem [30]. This chapter discusses methods of obtaining nodal prices.

5.2 Obtain Nodal Price By Solving OPF

The OPF problem in power system can be expressed as follows:

$$\begin{aligned} &\text{Minimize} && C(\mathbf{p}) \\ &\text{Subject to:} && \mathbf{F(x,p) = 0} \\ & && \mathbf{G(x,p) <= 0} \end{aligned} \tag{5.1}$$

where C(**p**) is the total cost, which is the objective function. F(**x,p**) is the active and reactive power balance equality constraint[1]. **G(x,p)** is the inequality constraint, including generator limits and line flow limits.

Assume the system has g transmission lines, n buses, including m generation buses and (n-m) load buses. In order to simplify the discussion, assume one generator at each generation bus, which can be easily generalized to multiple generators at each generation bus. Before the optimization process, the load flow of the system is solved, which means the steady state equilibrium point has been reached. (5.1) is a nonlinear optimization problem, and it is linearized at the equilibrium point, with the goal of minimizing the total cost. The optimization variables **x** is the generation change at each generator after optimization. Some key restrictions are:

1) Power balance must be kept all the time, which is the law of physics. This is the equality constraint.
2) The power flow on the transmission lines cannot exceed their thermal capacity.[2] This is the inequality constraint.
3) Power generation at each generator must not exceed the generation capacity, and no lower than zero. This sets up the upper and lower bound of the optimization variables.
4) Variable **x** can be any value between lower and upper boundaries.

The linearized OPF problem is as follows:

Minimize: $\mathrm{C} = \mathbf{c}^{\mathrm{T}}\,\mathbf{x}$

Subject to: $\mathbf{A}^{(1)}\,\mathbf{x} \le \mathbf{b}^{(1)}$

$$\mathbf{A}^{(2)}\,\mathbf{x} = \mathbf{b}^{(2)}$$

$$\mathbf{LB} \le \mathbf{x} \le \mathbf{UB}$$

$$\mathbf{b}^{(1)} \ge \mathbf{0},\ \mathbf{b}^{(2)} = \mathbf{0},\ \mathbf{LB} \le \mathbf{0},\ \mathbf{UB} \ge \mathbf{0}\,. \tag{5.2}$$

where: **x**: $\mathrm{m}\times 1$, generation change at each generator after optimization.

c: $\mathrm{m}\times 1$, bid price of each generator.

$\mathbf{A}^{(1)}$: $\mathrm{g}\times\mathrm{m}$, partial system PTDF matrix.[3]

[1] There are in reality two different formulations of the same problem. In one formulation the power balance equations is expressed as a simple one-equation constraint. In the expanded formulation, the full set of "load flow" power balance equations are used in the formulation.
[2] Other limits such as voltage limits or stability limits can, under some additional assumptions and with some caveats, also be expressed as flow limits.

$\mathbf{b}^{(1)}$: g×1, available transmission capacity in each transmission line.

$\mathbf{A}^{(2)}$: 1×m, reciprocals of penalty factors at each generator.

$\mathbf{b}^{(2)}$: 1×1, total change in generation after optimization, which is zero.

LB: m×1, lower boundary of **x**, which is the negative value of the generator power output. This means generator can reduce the generation to zero.

UB: m×1, upper boundary of **x**, which is the difference of the generation capacity and its power output. This means generator can only increase its generation to its capacity.

Form an "augmented" cost function or Lagrangian:

$$\boldsymbol{L} = \mathrm{C} - \lambda_{\mathrm{eq}} (\mathbf{A}^{(2)} \mathbf{x} - \mathbf{b}^{(2)}) - \lambda_{\mathrm{ineq}} (\mathbf{A}^{(1)} \mathbf{x} - \mathbf{b}^{(1)}) \qquad (5.3)$$

where: λ_{eq}: 1×1, the Lagrange multiplier of the equality constraint.

λ_{ineq}: 1×g, the Lagrange multiplier of the inequality constraint.

From $\frac{\partial \boldsymbol{L}}{\partial \mathrm{P_i}} = 0$, the nodal price at bus i (generator bus) is:

$$\mathrm{NP_i} = \frac{\partial \mathrm{C}}{\partial \mathrm{P_i}} = \lambda_{\mathrm{eq}}\, \mathbf{A}^{(2)}{}_{\mathrm{i}} + \lambda_{\mathrm{ineq}}\, \mathbf{A}^{(1)}(:,\mathrm{i}) \qquad (5.4)$$

where: $\mathrm{P_i}$ is the power consumed at bus i.

---

[3] PTDF: Power Transfer Distribution Factors, which express the sensitivity of specific flows to specific power injections relative to a "reference bus" location.

$\mathbf{A}^{(2)}{}_i$ is the ith element of row vector $\mathbf{A}^{(2)}$,

$\mathbf{A}^{(1)}(:,i)$ is the ith column of PTDF matrix $\mathbf{A}^{(1)}$.

In order to obtain nodal prices at all buses, next assume:

$\mathbf{A}^{(3)}$: g×n, is the full system PTDF matrix,

$\mathbf{A}^{(4)}$: 1×n, reciprocals of penalty factors at each bus.

Then:

$$\mathbf{NP} = \mathbf{A}^{(4)\,T}\,\lambda_{eq}{}^{T} + \mathbf{A}^{(3)\,T}\,\lambda_{ineq}{}^{T} \tag{5.5}$$

where: **NP**: n×1, the nodal price vector of all the n buses in the system.

Discussion

- There is a Lagrange multiplier associated with every constraint, both equality and inequality.
- Each constraint is associated with a vector of adjustment factors: λ_{eq} with $\mathbf{A}^{(2)}$, λ_{ineq} with $\mathbf{A}^{(1)}$.
- The price at each location is the sum of the products of the Lagrange multipliers with the associated adjustment factors.
- $\mathbf{A}^{(2)}{}_i = (1 - \frac{\partial P_L}{\partial P_i})$, with P_L representing the system transmission loss. Assume $\lambda_{ineq} = 0$, which means there are no constrained transmission lines, then

$$\frac{\partial C}{\partial P_i} = \lambda_{eq} \left(1 - \frac{\partial P_L}{\partial P_i}\right) \tag{5.6}$$

Consider penalty factor $L_i = \dfrac{1}{1 - \dfrac{\partial P_L}{\partial P_i}}$, an alternative expression for (5.6) is

$$\frac{\partial C}{\partial P_i} L_i = \lambda_{eq} \tag{5.7}$$

This is a classical economic dispatch result. When line losses are ignored, $\dfrac{\partial P_L}{\partial P_i} = 0$, the optimal dispatch rule is: if the incremental cost at each generator varies within a range, as in [35], then operate all generators, not at their limits, at equal incremental cost λ_{eq}. If the incremental cost at each generator is fixed (as bid price), then λ_{eq} is the bid price of the marginal generator, which supplies the next unit of power needed by the system.

When line losses are considered, the optimal dispatch rule is to operate all the generators so that the product $\dfrac{\partial C}{\partial P_i} L_i = \lambda_{eq}$ for every generator. It is obvious that a large penalty factor makes the corresponding generator less attractive and a lower nodal price from that generator is required, also generators remote from load centers will have larger penalty factors than generators that are close.

- $\mathbf{A}^{(1)}$ is the PTDF matrix of the system. As discussed in Section 4.2, PTDF (Power Transfer Distribution Factor) is the sensitivity of a flow to an injection. $\mathbf{A}^{(1)}(i,j)$

reflects the effect of the power injection at bus j to the power flow at transmission line i. From the Kuhn-Tucker Conditions in Section 4.5, if the flow on line No. m reached the constraint limit, then $\lambda_{ineq}(m) > 0$, with $m = 1, \ldots, g$. The value of $\lambda_{ineq}(m)$ means if the transmission capacity on line m is increased by 1 unit (for example, 1 MW), how much the total cost will decrease. If the flow on line No. n has not reached the limit, then $\lambda_{ineq}(m) = 0$, which means increase the transmission capacity will not affect the total cost.

- The Lagrange multipliers are also called shadow prices, fictitious prices, etc [42]. They act as the implicit prices or costs associated with the constraints. Price control can be performed by directly manipulating the Lagrange multipliers. Further discussion of this topic is included in Chapter 6.

5.3 Obtain Nodal Price By Approximate Method

The OPF based nodal price calculation method discussed above includes both losses and congestion effects. Losses can result in economic inefficiencies and increases the number of marginal units. However, most present-day nodal pricing systems ignore losses [38].

With losses ignored, an approximate nodal pricing calculation method can be used. This method can be expressed as [32]:

$$\text{Nodal Prices} = \text{PTDFs} + \text{Marginal Units} \tag{5.8}$$

The detail of this method is:

1. Identify the constraining conditions.

2. Identify the marginal units. There must be at least one more marginal unit than there are active constraints. The power for any location may come from more than one of the marginal units.

3. Solve a simple optimization problem. Find how cheap can power be delivered to a location from these units respecting all constrains.

The problem is a constrained optimization problem. However, once the marginal units and active constraints are known, the problem is reduced to the solution of a set of algebraic equations.

If losses are ignored, the approximation method discussed here and the OPF method discussed in section 5.2 give the identical results with identical constraining elements. When losses are considered, elements in $\mathbf{A}^{(2)}$ in (5.2) are no longer unity, the two methods will give different results.

The next section shows an example of calculating nodal prices using this method, also illustrates the relationship between the two methods discussed in section 5.2 and this section.

5.4 Comparison of Nodal Price Calculation Methods

Section 5.2 and Section 5.3 introduced the nodal price calculation algorithms based on OPF and the approximation method. In this section, the relations between these two methods are illustrated based on a simple example.

Assume the system swing bus is bus s. Also assume line j is congested, generators at bus i and bus j are the marginal generators, bus x is a bus in the system.

5.4.1 OPF Method

Section 5.2 introduced the formula of calculating nodal price based on solving the OPF problem, which is repeated here. The nodal price at bus i is:

$$NP_i = \frac{\partial C}{\partial P_i} = \lambda_{eq}\, \mathbf{A}^{(2)}{}_i + \lambda_{ineq}\, \mathbf{A}^{(1)}(:,i) \qquad (5.9)$$

where: P_i is the power consumed at bus i.

$\mathbf{A}^{(2)}{}_i$ is the ith element of row vector $\mathbf{A}^{(2)}$,

$\mathbf{A}^{(1)}(:,i)$ is the ith column of PTDF matrix $\mathbf{A}^{(1)}$.

If losses are ignored, $\mathbf{A}^{(2)}{}_i = 1$, (5.9) is modified as

$$NP_i = \frac{\partial C}{\partial P_i} = \lambda_{eq} + \lambda_{ineq}\, \mathbf{A}^{(1)}(:,i) \qquad (5.10)$$

Since only line j is congested, $\lambda_{ineq}(j) \neq 0$, all other elements of λ_{ineq} are zero. $\mathbf{A}^{(1)}$ is the Power Transfer Distribution Factor (PTDF) matrix of the system, $PTDF_{sj} = 0$. Section 4.2 discussed PTDF in detail. From (5.10):

$$NP_i = \lambda_{eq} + \lambda_{ineq}(j)\, PTDF_{ij} \qquad (5.11)$$

$$NP_s = \lambda_{eq} + \lambda_{ineq}(j)\, PTDF_{sj} \qquad (5.12)$$

Substitute (5.12) to (5.11) to cancel λ_{eq}:

$$NP_i = NP_s + \lambda_{ineq}(j)\,(PTDF_{ij} - PTDF_{sj}) \tag{5.13}$$

Similarly, one can get:

$$NP_k = NP_s + \lambda_{ineq}(j)\,(PTDF_{kj} - PTDF_{sj}) \tag{5.14}$$

$$NP_x = NP_s + \lambda_{ineq}(j)\,(PTDF_{xj} - PTDF_{sj}) \tag{5.15}$$

From (5.13) and (5.14), $\lambda_{ineq}(j)$ can be written as:

$$\lambda_{ineq}(j) = \frac{NP_i - NP_k}{PTDF_{ij} - PTDF_{kj}} \tag{5.16}$$

Substitute (5.16) in (5.15):

$$NP_x = \frac{NP_i(PTDF_{xj} - PTDF_{kj}) - NP_k(PTDF_{xj} - PTDF_{ij})}{PTDF_{ij} - PTDF_{kj}} \tag{5.17}$$

5.4.2 Approximate Method

Section 5.3 introduced an approximate method for calculating nodal prices. Consider the above example. Assume 1 unit of power is delivered to bus x, and line j flow is not increased. Suppose P_i and P_k are the power generation at generator i and generator k, which are the marginal units. The following equation shows these relations:

$$\begin{bmatrix} PTDF_{ij} - PTDF_{xj} & PTDF_{kj} - PTDF_{xj} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} P_i \\ P_k \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{5.18}$$

Solve (5.18), one get:

$$P_k = \frac{PTDF_{ij} - PTDF_{xj}}{PTDF_{ij} - PTDF_{kj}} \tag{5.19}$$

$$P_i = \frac{PTDF_{xj} - PTDF_{kj}}{PTDF_{ij} - PTDF_{kj}} \tag{5.20}$$

The nodal price at bus x is:

$$NP_x = NP_i \cdot P_i + NP_k \cdot P_k \tag{5.21}$$

Substituting (5.19) and (5.20) into (5.21):

$$NP_x = \frac{NP_i(PTDF_{xj} - PTDF_{kj}) - NP_k(PTDF_{xj} - PTDF_{ij})}{PTDF_{ij} - PTDF_{kj}} \tag{5.22}$$

5.4.3 Conclusion

This simple example illustrates the relations between the OPF method and the approximate method of calculating nodal prices. The following observations can be made:

- It is obvious that (5.17) and (5.22) are same. This means when losses are ignored, with the same congestion elements, the two methods give identical results.
- When losses are considered, (5.10) can no longer represent (5.9), the two methods will give different results.

In most present-day nodal pricing systems, losses are ignored. The approximate method is a reasonable simplification of nodal price calculation under these circumstances.[4]

5.5 Examples

The New England 16 bus test system is used to demonstrate the nodal prices. The system one-line diagram is shown in Figure 5.1. Bus 1 is the system swing bus. Buses 1, 4, 9 10, 14, 16 are generator buses. Buses 2, 3, 5, 6, 7, 8, 11, 12, 13,15 are load buses. All the lines are summarized in Table 5.1, the line resistances are assumed to be zero by default. The capacity of each generator is listed in Table 5.2. The load information is listed in Table 5.3. In this section, assume the system MVA base is 1 MVA, base voltage is 1 kV, and base current is 1 kA. The generator bid prices are listed in Table 5.4. Branch 3 (Bus 1 to Bus 7) rating is 3 MVA, and all other branch ratings are 2 MVA.

5.5.1 Example 1

In this example, the following three different cases are studied:

1) The base case as described above, which ignores branch resistances.
2) Branch resistances are not ignored. They are listed in Table 5.5.
3) The load at each load bus is increased by 20%, with loss ignored.

After running OPF, the nodal prices ($/MW) at each node for three cases are shown in Table 5.6, and generator MW output is listed in Table 5.7. The total generation of all the

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[4] In most cases losses can be reintroduced if the penalty factors are used in the approximate formulation to adjust the PTDFs.

generators equal to the total load. The branch load flow is listed in Table 5.8. Figure 5.2 shows the nodal prices of the three cases. Figure 5.3 is the close look of the nodal price at Bus 8, which is the highest among all the buses.

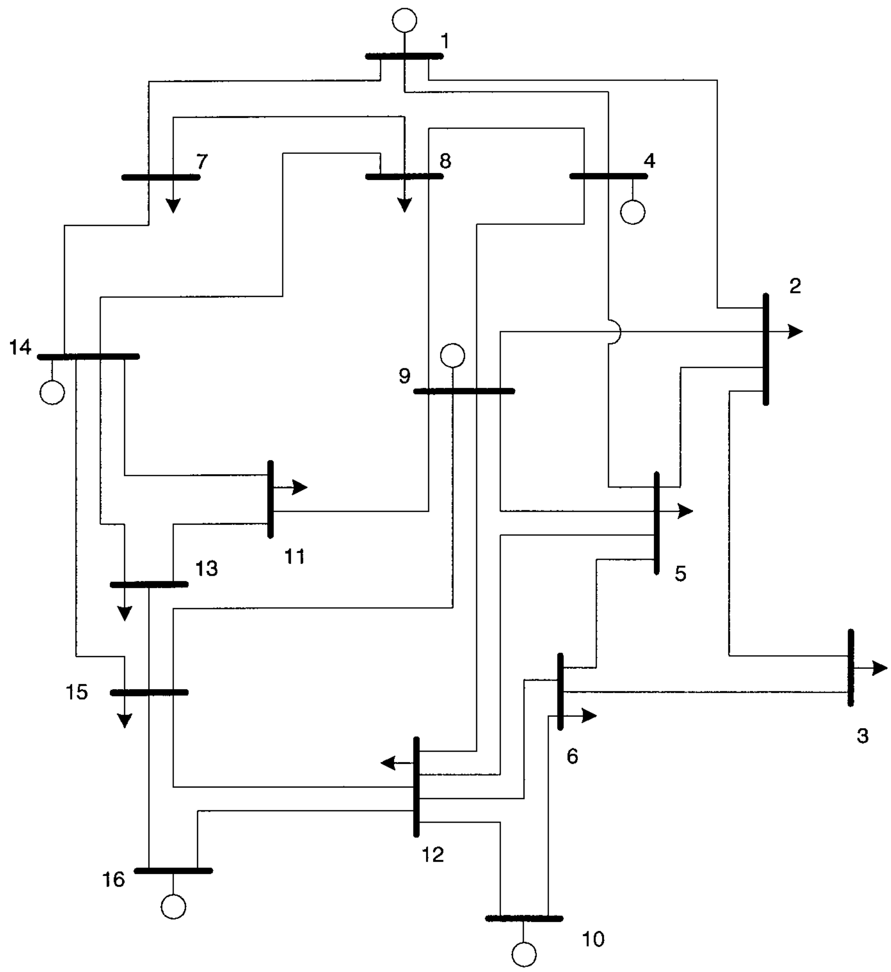

Figure 5.1: The New England 16 bus test system

Table 5.1: Branches in the Test System

Branch No.	From Bus No.	To Bus No.	Branch Impedance (pu)
1	1	2	0.0600i
2	1	4	0.0200i
3	1	7	0.0150i
4	2	3	0.0300i
5	2	5	0.0300i
6	2	9	0.0600i
7	3	6	0.0150i
8	4	8	0.0050i
9	4	5	0.0550i
10	4	9	0.0100i
11	5	6	0.0300i
12	5	9	0.0150i
13	5	12	0.0250i
14	6	10	0.0150i
15	6	12	0.0300i
16	7	8	0.0200i
17	7	14	0.0100i
18	8	9	0.0200i
19	8	14	0.0800i
20	9	11	0.0500i
21	9	12	0.0150i
22	9	15	0.0200i
23	10	12	0.0300i
24	11	13	0.0200i
25	11	14	0.0400i
26	12	15	0.0200i
27	12	16	0.0500i
28	13	14	0.0400i
29	13	15	0.0400i
30	14	15	0.0200i
31	15	16	0.0500i

Table 5.2: Generator Capacity

Generator Bus No.	1	4	9	10	14	16
Capacity (MW)	10	10	10	10	10	10

Table 5.3: Load Information

Load Bus No.	2	3	5	6	7	8	11	12	13	15
Load (MW)	1.5	1.5	1.5	1.5040	1.5	1.5	1.5	1.5	1.5	1.5

Table 5.4: Generator Bid Price

Generator Bus No.	1	4	9	10	14	16
Bid Price ($/MW)	5	4	7	3	9	8

Table 5.5: Branch Resistances for Case 2

Branch Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Branch R (pu)	0.0006	0.0008	0.0019	0.0016	0.0002	0.0003	0.0008	0.0012
Branch Number	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Branch R (pu)	0.0014	0.0027	0.0018	0.0010	0.0014	0.0018	0.0005	0.0025
Branch Number	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Branch R (pu)	0.0029	0.0018	0.0001	0.0024	0.0018	0.0021	0.0003	0.0013
Branch Number	**25**	**26**	**27**	**28**	**29**	**30**	**31**	
Branch R (pu)	0.0011	0.0005	0.0025	0.0025	0.0014	0.0029	0.0004	

Table 5.6: Bus Nodal Prices (NP) ($/MW): Example 1

Bus No.	Case 1	Case 2	Case 3
1	5.0000	5.0000	5.0000
2	6.8700	6.8900	6.8700
3	7.7600	7.8000	7.7600
4	4.0000	4.0000	4.0000
5	6.8500	6.8700	6.8500
6	8.2100	8.2400	8.2100
7	9.7500	9.7400	9.6500
8	**11.6400**	**11.6600**	**12.4900**
9	7.0000	7.0000	7.0000
10	3.0000	3.0000	3.0000
11	8.1800	8.2100	8.1800
12	6.7400	6.7600	6.7400
13	8.2500	8.2800	8.2500
14	9.0000	9.0000	9.0000
15	7.6200	7.6500	7.6200
16	7.1800	7.2000	7.1800

Table 5.7: Generator Output: Example 1

Bus No.	Case 1 (MW)	Case 2 (MW)	Case 3 (MW)
1	6.2652	6.4157	6.5578
4	2.8132	2.6792	2.5831
9	2.0797	1.9763	3.8915
10	2.6230	2.5257	2.4067
14	1.2229	1.4685	2.5657
16	0.0000	0.0000	0.0000
Total Generation	**15.0040**	**15.0653**	**18.0048**

Table 5.8: Branch MVA Flow: Example 1

Branch No.	Case 1	Case 2	Case 3
1	1.3862	1.4362	1.5673
2	1.8900	1.9751	**2.0000**
3	2.9999	2.9997	**3.0000**
4	0.5906	0.6063	0.7458
5	0.3135	0.3028	0.3957
6	0.5209	0.5205	0.6743
7	0.9879	0.9682	1.1153
8	**2.0000**	**2.0000**	**2.0000**
9	0.7102	0.7356	0.8091
10	1.9999	**2.0000**	1.7820
11	0.3529	0.3638	0.5285
12	1.3773	1.4119	1.8530
13	0.2229	0.2226	0.2236
14	**2.0000**	**2.0000**	**2.0000**
15	0.3382	0.3565	0.5323
16	0.2267	0.2650	0.3055
17	1.3893	1.2723	0.9901
18	0.5611	0.5756	0.4672
19	0.2594	0.2483	0.2282
20	0.7357	0.7010	0.8309
21	1.2153	1.2276	1.7179
22	0.9707	0.9355	1.1548
23	0.6879	0.6625	0.4718
24	0.2013	0.1961	0.2134
25	0.9172	0.9542	1.1326
26	0.2679	0.2596	0.2880
27	0.3164	0.3158	0.3170
28	0.9700	1.0137	1.1961
29	0.5058	0.4888	0.5441
30	0.9187	0.9989	1.3101
31	0.2962	0.2961	0.2970

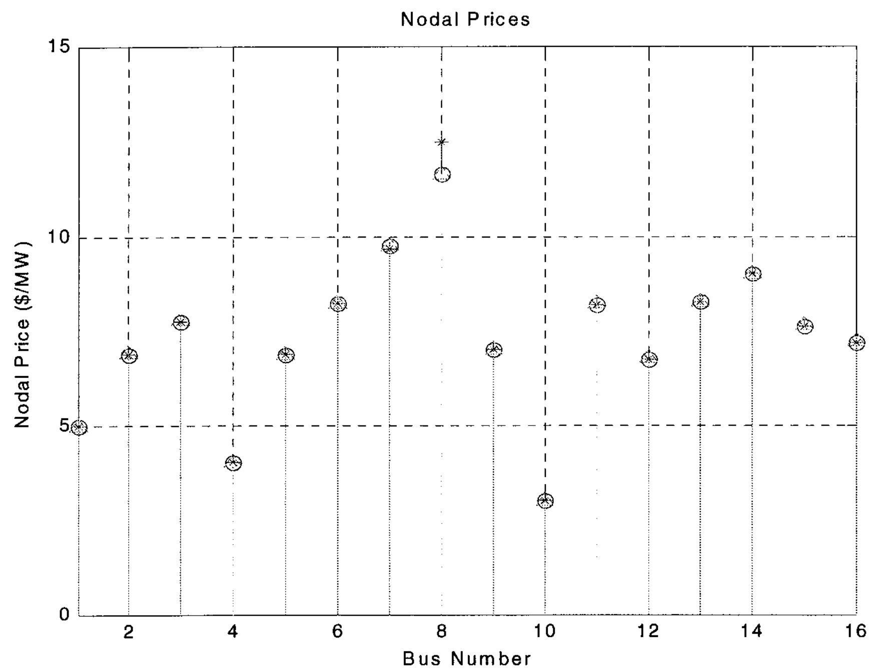

Figure 5.2: Nodal prices for different cases

(Circle: Case 1; Triangle: Case 2; Star: Case3)

Figure 5.3: Nodal price at Bus 8 (Circle: Case 1; Triangle: Case 2)

Several observations can be made:

- If transmission losses are considered, the nodal prices at most buses increase. The nodal prices become higher with higher losses.
- Transmission losses increase the total generation, but this does not mean all generators will increase generation, as shown in Table 5.7. Also the power flow on most transmission lines increased, more lines reached their limit.
- When losses are too high, the OPF solution shows that in order to supply all the loads, some lines become overloaded. This result suggests the losses can have a significant effect on system operation, and that losses should not be ignored.
- Load increases have similar effects on the system as transmission losses. The total generation increases, more lines reach their limit. When loads are too high, some lines become overloaded.

5.5.2 Example 2

In this example, the following three modifications (Case 4 to Case 6) of the base case (Case 1 in section 5.5.1) are studied:

4) Loss of generator at Bus 10.

5) Double the transmission capacity from Bus 4 to Bus 8 from 2 MVA to 4 MVA.

6) Decrease load at bus 8 from 1.5 MW to 0.5 MW.

After running OPF, the nodal prices ($/MW) at each node for three cases are shown in Table 5.9, and generator MW output is listed in Table 5.10. The total generation of all the generators equal to the total load. The branch load flow is listed in Table 5.11. Figure 5.4 shows the nodal prices of the three modifications.

Table 5.9: Bus Nodal Prices (NP): Example 2

Bus No.	Case 4	Case 5	Case 6
1	5.0000	4.9500	5.0000
2	6.7000	6.6700	6.6800
3	7.1700	7.3800	7.4000
4	4.0000	4.0000	4.0000
5	6.9400	6.6400	6.6500
6	7.4000	7.7400	7.7600
7	9.5600	5.2200	5.3300
8	**11.1000**	**4.7400**	**4.9600**
9	7.0000	7.0000	7.0000
10	7.5900	3.0000	3.0000
11	8.2600	6.1500	6.2000
12	7.9600	6.2900	6.3000
13	8.3900	6.0600	6.1100
14	9.0000	5.6500	5.7300
15	8.0400	6.2800	6.3100
16	8.0000	6.2800	6.3000

Table 5.10: Generator Output: Example 2

Bus No.	Case 4 (MW)	Case 5 (MW)	Case 6 (MW)
1	6.4290	0.0000	4.1333
4	2.9723	7.7460	4.0892
9	3.7644	4.7053	3.1728
10	0.0000	2.5527	2.6087
14	1.3979	0.0000	0.0000
16	0.4404	0.0000	0.0000
Total Generation	**15.0040**	**15.0040**	**14.0040**

Table 5.11: Branch MVA Flow: Example 2

Branch No.	Case 4	Case 5	Case 6
1	1.5494	0.5512	1.0850
2	1.8900	1.5109	0.6829
3	2.9999	0.9695	2.4019
4	0.9286	0.4405	0.5387
5	0.3400	0.6340	0.4182
6	0.6588	0.7709	0.6112
7	0.6331	1.1632	1.0462
8	**2.0000**	**3.4389**	**1.9999**
9	0.8676	0.8216	0.7558
10	1.9999	1.9999	1.9999
11	0.8741	0.4375	0.3870
12	1.9189	1.7583	1.5316
13	0.2412	0.2219	0.2215
14	0.7314	**2.0000**	**2.0000**
15	0.7839	0.3986	0.3502
16	0.2267	1.3700	0.6722
17	1.3893	0.8596	1.5683
18	0.5611	0.2966	0.5611
19	0.2594	0.4908	0.4159
20	0.7454	1.1170	0.9452
21	1.9999	1.6996	1.4444
22	1.1097	1.6591	1.3413
23	0.5068	0.6177	0.6736
24	0.2133	0.2523	0.2263
25	0.9290	0.6345	0.7581
26	0.4631	0.4707	0.3720
27	0.4322	0.3255	0.3205
28	0.9924	0.7236	0.8310
29	0.4635	0.6785	0.6036
30	1.0577	0.0279	0.4142
31	0.3285	0.3060	0.3006

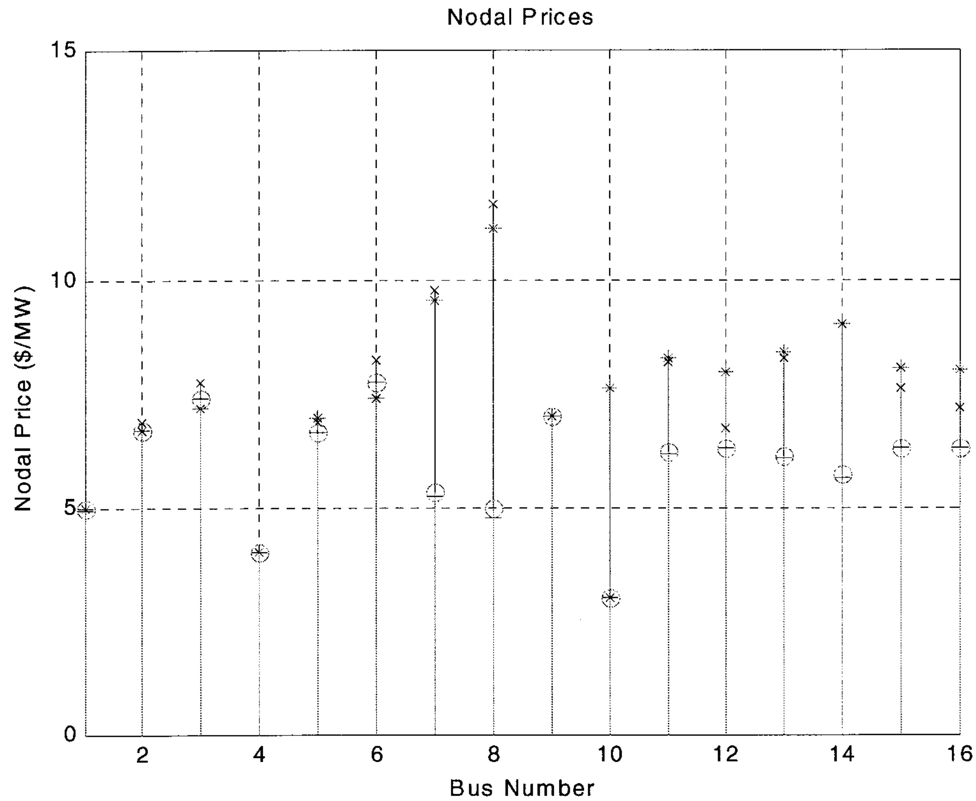

Figure 5.4: Nodal prices for different modifications

(x-mark: Case 1; Star: Case 4; Plus: Case 5; Circle: Case 6)

Several observations can be made:

- The loss of the generator at Bus 10, which has the lowest bidding price, increases the nodal prices at Buses 10, 12 15, 16, but decreases the nodal prices at Buses 2, 3, 6, 7, 8.
- By increasing the transmission capacity from Bus 4 to Bus 8, the nodal price at Bus 8 is lowered. This case demonstrates that plenty of transmission capacity is very important in relieving transmission congestion.
- By decreasing load from 1.5 MW to 0.5 MW at Bus 8, the nodal price at Bus 8 is lowered again. This case shows that active demand side load management is also very important in relieving transmission congestion.
- Among case 1 to case 3, the nodal price *patterns* are very similar. Case 4 to case 6 show that if the system changes significantly, such as adding or losing a major transmission line, a big generation plant, or a major load, etc., the nodal price pattern of the whole system changes significantly. This is an important observation, which will become the theoretical fundamental for the systematic zonal separation method described in Chapter 7.

Chapter 6

Effect of Price Caps on the Nodal Price

6.1 Introduction

A very important and interesting problem is how the price caps affect the power flow and nodal prices under all the constraints of the system. This chapter establishes the Optimal Power Flow (OPF) model under price cap and presents some experimental results based on the 16-bus test system.

6.2 Duality Theory of Linear Programming

Linear Programming is used extensively in the optimization problems. For example, the linearized OPF problem in Chapter 5 is solved by MATLAB using Linear Programming method. For every linear programming problem, there is a companion problem, called the dual linear problem [48], in which the roles of variables and constraints are reversed. That is, for every variable in the original or "primal" linear programming problem, there is a corresponding constraint in the dual problem; and for every constraint in the primal, there is also a corresponding variable in the dual [48].

The variables in the dual problems (dual variables) are the Lagrange multipliers of the primal problem. One of the effects of duality theory is to make explicit the effect of changes in the constraints on the value of the objective [48]. The dual variables are often called "accounting prices", "fictitious prices", "shadow prices", and "imputed values" [42, 49], because they measure the implicit "costs" associated with the constraints. The related complementarity problems are discussed in [50]. Reference [51] discussed the engineering and economic applications of complementarity problems.

Dual linear program has been used to study price caps in other fields of economic study. For example, in reference [50], price supports and price ceilings for the transportation problem are incorporated into the dual linear program, and then the modified mixed complementarity problem is formed. But no such application has been found in the power system literature.

Chapter 5 discussed the method of calculating nodal prices by using OPF. When the price caps are applied, adding additional constraints, which correspond to the price caps, on the dual variables would modify the dual problem. After solving the dual of the modified dual problem, which is the modified primal problem, one can find how the price caps affect power flows, nodal prices, and other characteristics of the power system markets.

6.3 Equation Derivation

The nodal price calculation method in Chapter 5 is repeated here. Assume the system has g transmission lines, n buses, including m generation buses and (n-m) load buses. Rewrite (5.2) as:

Minimize: $C = \mathbf{c}^T \mathbf{x}$

Subject to: $\mathbf{A}^{(1)} \mathbf{x} \le \mathbf{b}^{(1)}$

$\mathbf{A}^{(2)} \mathbf{x} = \mathbf{b}^{(2)}$

$\mathbf{LB} \le \mathbf{x} \le \mathbf{UB}$

$$\mathbf{b}^{(1)} \ge \mathbf{0},\ \mathbf{b}^{(2)} = \mathbf{0},\ \mathbf{LB} \le \mathbf{0},\ \mathbf{UB} \ge \mathbf{0} \tag{6.1}$$

where: $\mathbf{x}$: m×1, generation change at each generator after optimization.

$\mathbf{c}$: m×1, bid price of each generator.

$\mathbf{A}^{(1)}$: g×m, system PTDF matrix.

$\mathbf{b}^{(1)}$: g×1, available transmission capacity in each transmission line.

$\mathbf{A}^{(2)}$: 1×m, reciprocals of penalty factors at each generator.

$\mathbf{b}^{(2)}$: 1×1, total change in generation after optimization, which is zero.

LB: m×1, lower boundary of **x**, which is the negative value of the generator power output. This means generator can reduce the generation to zero.

UB: m×1, upper boundary of **x**, which is the difference of the generation capacity and its power output. This means generator can only increase its generation to its capacity.

Modify the primal problem:

Minimize: $C = \mathbf{c}^T \mathbf{x}$

Subject to: $\mathbf{A}^{(1)} \mathbf{x} \leq \mathbf{b}^{(1)}$

$$-\mathbf{I}\,\mathbf{x} \leq -\mathbf{LB}$$

$$\mathbf{I}\,\mathbf{x} \leq \mathbf{UB}$$

$$\mathbf{A}^{(2)} \mathbf{x} = \mathbf{b}^{(2)}$$

$$\mathbf{b}^{(1)} \geq \mathbf{0},\ \mathbf{b}^{(2)} \geq \mathbf{0},\ \mathbf{LB} \leq \mathbf{0},\ \mathbf{UB} \geq \mathbf{0}\,. \tag{6.2}$$

where: $\mathbf{I}$: m×m.

The dual problem of the modified primal problem is:

Maximize: $$C' = [\mathbf{b}^{(1)T} \quad -\mathbf{LB}^T \quad \mathbf{UB}^T \quad \mathbf{b}^{(2)T}] \begin{bmatrix} \mathbf{w}^{(1)} \\ \mathbf{w}^{(2)} \\ \mathbf{w}^{(3)} \\ \mathbf{w}^{(4)} \end{bmatrix}$$

Subject to:

$$[\mathbf{A}^{(1)T} \quad -\mathbf{I}^T \quad \mathbf{I}^T \quad \mathbf{A}^{(2)T}] \begin{bmatrix} \mathbf{w}^{(1)} \\ \mathbf{w}^{(2)} \\ \mathbf{w}^{(3)} \\ \mathbf{w}^{(4)} \end{bmatrix} = \mathbf{c}$$

$$\mathbf{w}^{(1)} \leq \mathbf{0},\ \mathbf{w}^{(2)} \leq \mathbf{0},\ \mathbf{w}^{(3)} \leq \mathbf{0},\ \mathbf{w}^{(4)} \text{ unrestricted.} \tag{6.3}$$

where: $\mathbf{w}^{(1)}$: g×1, the Lagrange multipliers of the inequality constraints.

$\mathbf{w}^{(2)}$: m×1, the Lagrange multipliers of the lower bound of **x**.

$\mathbf{w}^{(3)}$: m×1, the Lagrange multipliers of the upper bound of **x**.

$\mathbf{w}^{(4)}$: 1×1, the Lagrange multipliers of the equality constraint.

The nodal price (**NP**) for each generator is calculated in (5.5), which is repeated as:

$$\mathbf{NP} = \mathbf{A}^{(4)\,T} \lambda_{eq}^{\,T} + \mathbf{A}^{(3)\,T} \lambda_{ineq}^{\,T} \tag{6.4}$$

where: **NP**: n×1, the nodal price vector of all the n buses in the system.

$\mathbf{A}^{(3)}$: g×n, is the full system PTDF matrix,

$\mathbf{A}^{(4)}$: 1×n, reciprocals of penalty factors at each bus.

λ_{eq}: 1×1, the Lagrange multiplier of the equality constraint, $\lambda_{eq}^{\,T} = \mathbf{w}^{(4)}$.

λ_{ineq}: 1×g, the Lagrange multiplier of the inequality constraint, $\lambda_{ineq}^{\,T} = \mathbf{w}^{(1)}$.

When the price cap is applied,

$\mathbf{NP} \le \mathbf{K}$,

where: **K**: m×1 constant vector, each element is k, the price cap.

From (6.4):

$$\mathbf{A}^{(4)\,T} \mathbf{w}^{(4)} + \mathbf{A}^{(3)\,T} \mathbf{w}^{(1)} \le \mathbf{K} \tag{6.5}$$

The modified dual problem is:

$$\text{Maximize: } C' = [\mathbf{b}^{(1)T} \; \mathbf{-LB}^{T} \quad \mathbf{UB}^{T} \quad \mathbf{b}^{(2)T}] \begin{bmatrix} \mathbf{w}^{(1)} \\ \mathbf{w}^{(2)} \\ \mathbf{w}^{(3)} \\ \mathbf{w}^{(4)} \end{bmatrix}$$

Subject to

$$[\mathbf{A}^{(1)T} \quad \mathbf{-I}^{T} \quad \mathbf{I}^{T} \quad \mathbf{A}^{(2)T}] \begin{bmatrix} \mathbf{w}^{(1)} \\ \mathbf{w}^{(2)} \\ \mathbf{w}^{(3)} \\ \mathbf{w}^{(4)} \end{bmatrix} = \mathbf{c}$$

$$[\mathbf{A}^{(1)T} \quad \mathbf{0_1}^{T} \quad \mathbf{0_1}^{T} \quad \mathbf{A}^{(2)T}] \begin{bmatrix} \mathbf{w}^{(1)} \\ \mathbf{w}^{(2)} \\ \mathbf{w}^{(3)} \\ \mathbf{w}^{(4)} \end{bmatrix} \leq \mathbf{K}$$

$$\mathbf{w}^{(1)} \leq \mathbf{0}, \ \mathbf{w}^{(2)} \leq \mathbf{0}, \ \mathbf{w}^{(3)} \leq \mathbf{0}, \ \mathbf{w}^{(4)} \text{ unrestricted.} \qquad (6.6)$$

where: $\mathbf{0_1}$: m×m all zero matrix.

The primal problem of (6.6) is:

$$
\begin{aligned}
\text{Minimize:}\quad & C = \mathbf{c}^T \mathbf{x} + \mathbf{K}^T \mathbf{x'} \\
\text{Subject to:}\quad & \mathbf{A}^{(1)} (\mathbf{x} + \mathbf{x'}) \leq \mathbf{b}^{(1)} \\
& \mathbf{A}^{(2)} (\mathbf{x} + \mathbf{x'}) = \mathbf{b}^{(2)} \\
& \mathbf{LB} \leq \mathbf{x} \leq \mathbf{UB} \\
& \mathbf{x'} \geq \mathbf{0} \\
& \mathbf{b}^{(1)} \geq \mathbf{0},\ \mathbf{b}^{(2)} \geq \mathbf{0},\ \mathbf{LB} \leq \mathbf{0},\ \mathbf{UB} \geq \mathbf{0}\,. \qquad (6.7)
\end{aligned}
$$

where: $\mathbf{x}$: m×1, $\mathbf{x'}$: m×1, $\mathbf{c}$: m×1, $\mathbf{A}^{(1)}$: g×m, $\mathbf{b}^{(1)}$: g×1, $\mathbf{A}^{(2)}$: 1×m, $\mathbf{b}^{(2)}$: 1×1,

$\mathbf{LB}$: m×1, $\mathbf{UB}$: m×1.

Comparing (6.1) and (6.7), one can see that the application of price cap introduces extra generation **x'** at every generator bus, and this generation does not necessarily come from the local generators. The marginal cost of **x'** is the price cap. The combination of the original generation of the generators **x** and the extra generation **x'** in (6.7) should subject to the same equality and inequality constraints as in (6.1). The application of above derivation in the 16-bus system is discussed in the following section.

6.4 Examples

The New England 16 bus test system described in Chapter 5 is used to demonstrate the effect of price caps on the system. The system one-line diagram was shown in Figure 5.1. Bus 1 is the system swing bus. The summary of all the branches was listed in Table 5.1.

In the examples discussed in this section, it is assumed that generator bid prices do not change with the price caps. This assumption simplifies the study, and concentrates on the effect of price caps on the generation and load.

6.4.1 Example 1

In this example, the generator bid prices are as following:

Table 6.1: Generator Bid Price: Example 1

Generator Bus No.	1	4	9	10	14	16
Bid Price ($/MW)	5	4	7	3	9	8

Branch 3 (Bus 1 to Bus 7) rating is 3 MVA, all other branch ratings are 2 MVA. After running OPF, the nodal prices ($/MW) at each node are shown in Table 6.2, and generator MW output is listed in Table 6.3. Nodal Price at Bus 8 is the highest at $11.64/MW. The total generation from all the generators equals to the total load 15.0040 MW.

Table 6.2: Bus Nodal Prices (No Price Cap): Example 1

Bus #	1	2	3	4	5	6	7	8
Nodal Price ($/MW)	5.00	6.87	7.76	4.00	6.85	8.21	9.75	**11.64**
Bus #	9	10	11	12	13	14	15	16
Nodal Price ($/MW)	7.00	3.00	8.18	6.74	8.25	9.00	7.62	7.18

Table 6.3: Generator MW output (No Price Cap): Example 1

Generator Bus No.	1	4	9	10	14	16	**Total MW**
Power (MW)	6.2652	2.8132	2.0797	2.6230	1.2229	0	**15.0040**

Next, price cap of $11/MW is applied to the system, after running the modified OPF as in (6.7), the new nodal prices are in Table 6.4, which shows no nodal price is higher than $11/MW. Generator MW output is listed in Table 6.5. The total generator output is 14.4074 MW, which is less than the total load. The modified OPF results suggest that 0.5966 MW power is needed at Bus 8 to balance power and solve the problem, or if reduce load at Bus 8 by 0.5966 MW, the problem can be solved as well.

This example shows that when the load buses have the highest nodal price, inappropriate (too-low) price cap requires load shedding, or adding local generation (Distributed Generation) at these buses. Otherwise, some transmission lines will be overloaded.

Table 6.4: Bus Nodal Prices (Price Cap Applied): Example 1

Bus #	1	2	3	4	5	6	7	8
Nodal Price ($/MW)	5.00	6.85	7.73	4.00	6.83	8.17	9.33	**11.00**
Bus #	9	10	11	12	13	14	15	16
Nodal Price ($/MW)	7.00	3.00	7.99	6.70	8.04	8.69	7.50	7.10

Table 6.5: Generator MW output (Price Cap Applied): Example 1

Generator Bus No.	1	4	9	10	14	16	**Total MW**
Power (MW)	5.7306	3.2664	2.7805	2.6299	0	0	**14.4074**

6.4.2 Example 2

In this example, the generator bid prices are as following:

Table 6.6: Generator Bid Price: Example 2

Generator Bus No.	1	4	9	10	14	16
Bid Price ($/MW)	5	5.4	5.8	6	6.5	5.5

Branch 3 (Bus 1 to Bus 7) rating is 3.6 MVA, and all other branch ratings are 2.4 MVA. After running OPF, the nodal prices ($/MW) at each node are shown in Table 6.7, and generator MW output is listed in Table 6.8. Nodal Price at Bus 9 is the highest at $5.8/MW. The total generation from all the generators equal to the total load. Notice a generator is located at Bus 9, which has bid price $5.8/MW. This means the nodal price at Bus 9 is the bid price of the generator located there.

Table 6.7: Bus Nodal Prices (No Price Cap): Example 2

Bus #	1	2	3	4	5	6	7	8
Nodal Price ($/MW)	5.00	5.58	5.65	5.40	5.69	5.68	5.32	5.45
Bus #	9	10	11	12	13	14	15	16
Nodal Price ($/MW)	**5.80**	5.69	5.59	5.72	5.57	5.46	5.63	5.50

Table 6.8: Generator MW output (No Price Cap): Example 2

Generator Bus No.	1	4	9	10	14	16	**Total MW**
Power (MW)	7.5310	2.8289	0.0090	0	0	4.6351	**15.0040**

Next, price cap of \$5.75/MW is applied to the system. After running the modified OPF as in (6.7), the new nodal prices are in Table 6.9, which shows no nodal price is higher than \$5.75/MW. Generator MW output is listed in Table 6.10. The new total generation (14.9957 MW) is less than the total load. The modified OPF results suggest that extra 0.0083 MW power, no matter the price, is needed at Bus 9 to keep system power balance. No generator in the system can generate this extra 0.0083 MW without violating constraints.

Table 6.9: Bus Nodal Prices (Price Cap Applied): Example 2

Bus #	1	2	3	4	5	6	7	8
Nodal Price (\$/MW)	5.00	5.54	5.61	5.40	5.65	5.64	5.30	5.44
Bus #	9	10	11	12	13	14	15	16
Nodal Price (\$/MW)	**5.75**	5.65	5.56	5.67	5.54	5.44	5.60	5.50

Table 6.10: Generator MW output (Price Cap Applied): Example 2

Generator Bus No.	1	4	9	10	14	16	**Total MW**	**Additional Power Needed at Bus 9**
Power (MW)	7.5313	2.8288	0	0	0	4.6356	**14.9957**	**0.0083**

This example shows that when the generator buses have the highest nodal prices, inappropriate (too-low) price cap, which is lower than the operating cost of these generators, will force the generators at these buses to shut down. In order to balance the system, extra power from outside the system is needed at these buses, no matter what the cost is. Strategically placed Distributed Generation (DG) at these buses can also solve the problem. Otherwise, some transmission lines will be overloaded.

Another interpretation is that when the price caps are too low in one region, the generators inside the region may opt to sell power to entities outside of the region. To resolve the resulting deficiencies for consumers who cannot buy power at prices below the cap, the system operator had to buy power from outside sources, at the prevailing market-clearing price, which is usually higher than the price caps[1]. This sets up an obvious strategy by generators to sell power out of the region and then resell it back in to the region at above the cap.

[1] Of course, instead of "buying" the power, rules could be in place that empower the system operator to "mandate" or "order" that power be produced by certain generators at the capped price or some other pre-established price, thereby bypassing the markets entirely. The counterpart to this on the demand side is the implicit authority all operators have to "shed load" in cases of emergency, which can also be used to help attain price caps.

Chapter 7

Methodology for Automatic Zone Creation/Merging/Partition

7.1 Introduction

When congestion occurs in a path that completely splits two parts of the system, determination of zonal prices is trivial: each zone has its own unique zonal price.[1] Many "paths" are part of nomograms and the limits on these paths are a function of the flows on the individual path components. In a few cases, as for example in the case of paths that represent stability limits, the variable that is constrained is the sum of powers in across the path lines. In other cases, nomograms define more complex functions of these flows as constraining elements. In almost all cases, however, a path does not completely split the system unless external loop flows are ignored. Path constraints that rely on stability or

[1] In the presence of losses it is possible that the prices within a zone could be differentiated according to their impact on losses as established by their respective penalty factors.

other such "sum of all cutset flows" limits tend to produce good but not complete system separation. On many cases the limits are not on an entire "cutset" path but on one or more path components or on some combination of the individual path flows, perhaps augmented with some voltage or other such limits. Under these conditions, there can be a great deal of price dispersion among individual nodes in the network. There may be a continued desire for other reasons to nevertheless use the notion of a path that splits the network and thus retain the notion of unique zonal prices. However, it is of interest in this work to *not* start from this presumption but to allow individual lines or arbitrary combination of lines to become congested. Only after individual nodal prices are determined, and starting from these disperse nodal prices, zones are defined. After zones are defined, zonal prices can be subsequently determined by aggregation.

7.2 Methodology for Automatic Zone Creation

As discussed in Section 6.4, the two nodal price calculation methods (algorithms based on OPF and the approximation method) give identical results when losses are ignored. In most present-day nodal pricing systems, losses are ignored. The approximate method is a reasonable simplification of nodal price calculation under these circumstances. In the development of the automatic zone creation algorithm in this chapter, the approximation method is used to calculate the nodal prices in California system.

Network structure, such as where the loads and generators are located, the size of load and generation, the capacity of transmission lines, etc., determine where and when congestion happens. Every congestion pattern corresponds to a network structure. As demonstrated in the 16-bus example in section 5.5, the nodal price patterns are very similar between different operating conditions, unless the system changes significantly, such as adding or losing a major transmission line, a big generation plant, or a major load, etc.

In this section, for each case considered, it is assumed that only single line is congested, all other transmission lines have high capacity and will not congest. The purpose of making this simplification is to easily show that the location and the price differentials of the re-dispatching generating units have little to do with the resulting zonal pattern, and the pattern is only determined by the network structure. (Note: although changing the location and the price differentials of the marginal generating units can lead to different price differentials in nodal prices, the patterns of price variation stay fundamentally the same). The same conclusion can be drawn from the more realistic scenarios with multiple congestion lines.

The proposed methodology functions as follows:

- Potentially congested lines are identified as follows: (a) every line that is part of a path definition, and (b) any line that has congested at some time in the past leading to the need for re-dispatch.
- Potentially marginal generators are identified as any generator that (a) routinely submits inc (increment) and dec (decrement) bids, (b) has participated as part of prior RMR (Reliability Must Run) conditions, (c) is part of the OOM (Out Of Market) set, or (d) is part of the OOS (Out Of Service) set.
- For every individual potentially congested line (as defined above), one or more pairs of re-dispatchable units are identified from among the generators that are chosen above.

- Using these lines and locations, the appropriate PTDFs are determined for all system locations and for the lines that can potentially congest.
- For every marginal generator pair selected, a set of price differentials will be assumed for the redispatch. When no other information is available, it will be assumed that there is a 20% price differential between the two units, arranged as a ±10% from the base price conditions. (Note: the results are largely independent of this assumption, in that the assumption amounts to nothing more than a scaling of the price difference patterns that result).
- Using these units as new marginal units in the system, pseudo-nodal-prices are computed for every node in the system. These prices will be the nodal price departures from the base price at every node.
- If the prices observed are clearly partitioned into two sets with (nearly) equal prices in each set, the network has been split and the path has been identified. Otherwise, the largest price differential across any line pair is identified. An important by-product of this step will be the "price spread" within each zone.
- If the system has not been completely split, the prices observed are classified into two or more (but not exceeding 20) price "bands" and zones are defined. The maximum number of zones defined by any one constraint is generally much lower than this heuristically chosen number of 20.
- After this has been done, an investigation of possible mergers and consolidations of zones can be performed for all those cases where trivial zones are identified (either

very small zones, or zones that did not lead to significantly different prices from other zones under most or all conditions).

7.3 Case Study on Path 26

Path 26 is a recognized WSCC transmission path, and part of the ISO controlled grid, which consists of three parallel 500 KV transmission lines between PG&E's Midway and Southen California Edison (SCE)'s Vincent Substations. Both ends of path 26 were located within the original SP15 congestion zone (south of Path 15). Therefore, path 26 was an SP15 intra-zonal interface. In this study, the original SP15 zone is considered

First, suppose when congestion happens, the bus with lowest PTDF has nodal price of $80/MW, the bus with highest PTDF has nodal price of $120/MW. The zonal breakup method proposed in Section 7.1 is implemented. Figure 7.1 to Figure 7.6 show three pairs of figures.

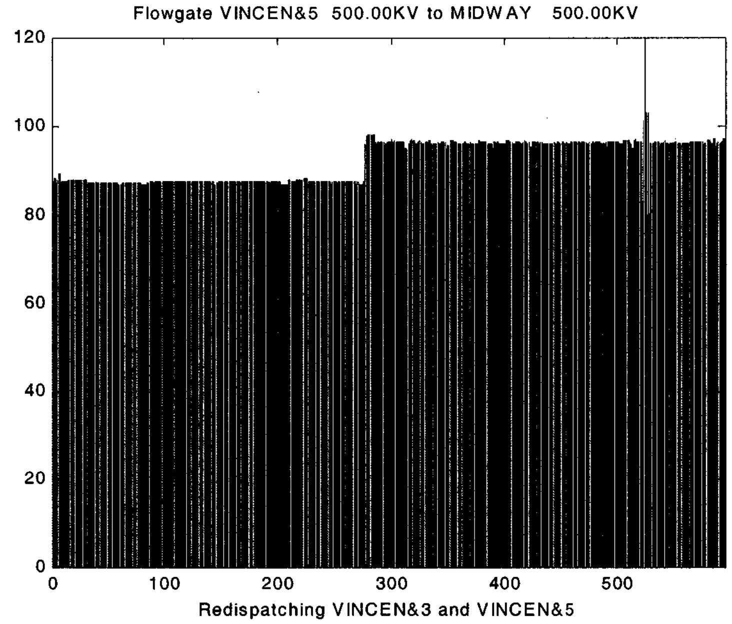

Figure 7.1: Nodal price patterns for flowgate from bus "Vincen&5" to bus "Midway"

(X axis: Bus Number; Y axis: Nodal Price ($/MW))

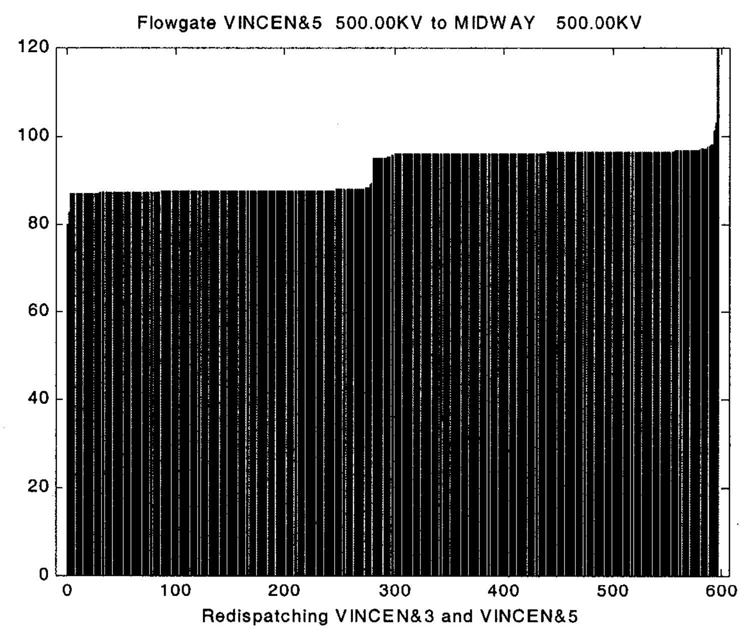

Figure 7.2: Sorted nodal price pattern for flowgate from bus "Vincen&5" to bus "Midway"

(X axis: Sorted Bus Number; Y axis: Nodal Price ($/MW))

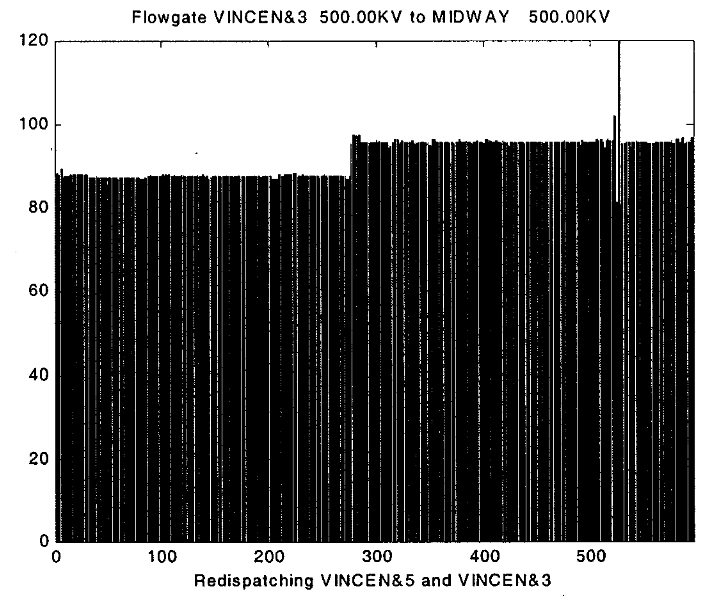

Figure 7.3: Nodal price patterns for flowgate from bus "Vincen&3" to bus "Midway"

(X axis: Bus Number; Y axis: Nodal Price ($/MW))

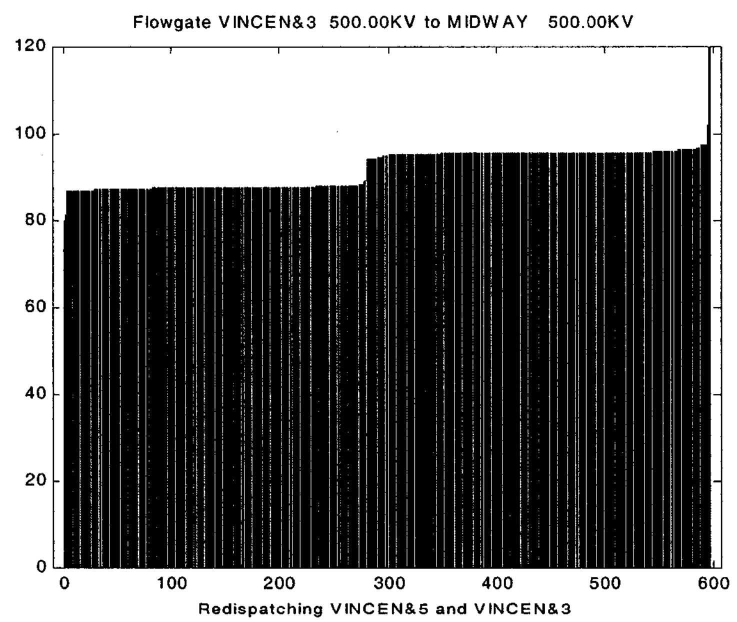

Figure 7.4: Sorted nodal price pattern for flowgate from bus "Vincen&3" to bus "Midway"

(X axis: Sorted Bus Number; Y axis: Nodal Price ($/MW))

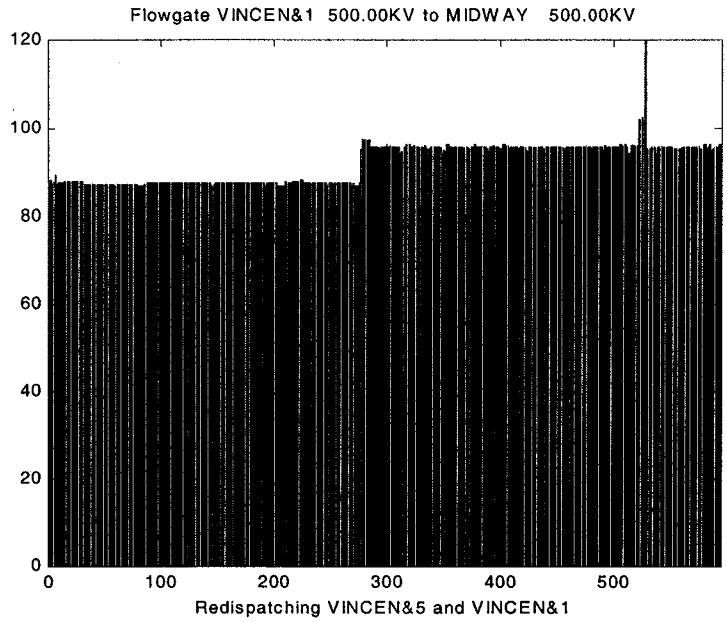

Figure 7.5: Nodal price patterns for flowgate from bus “Vincen&1” to bus “Midway”

(X axis: Bus Number; Y axis: Nodal Price ($/MW))

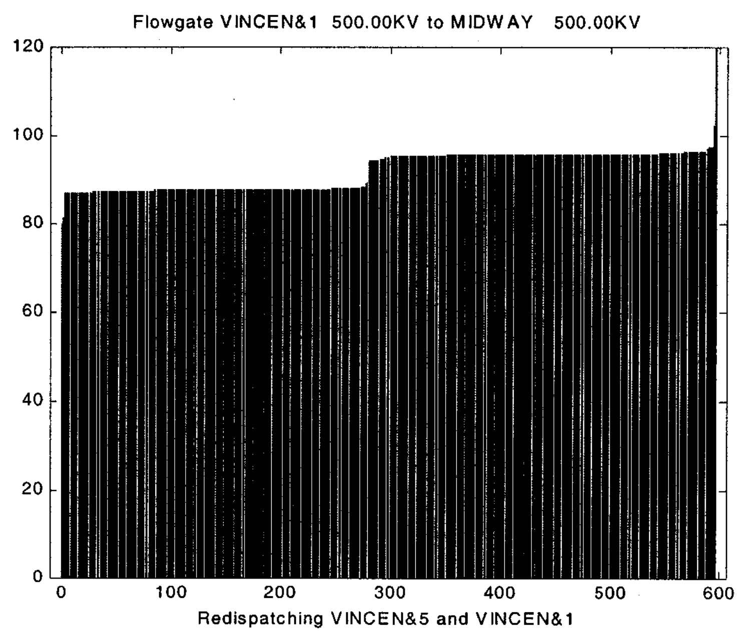

Figure 7.6: Sorted nodal price pattern for flowgate from bus “Vincen&1” to bus “Midway”

(X axis: Sorted Bus Number; Y axis: Nodal Price ($/MW))

It is obvious that the price patterns for the three lines of PATH 26 are very similar. The nodal prices are clearly separated into two zones except some high price "hot spots" and low price "cold spots". Intra-zonal congestion when the sum of the power across the path is congested often lead to a reasonable zonal structure except for the common presence of frequent "hot spots." These hot spots correspond to locations that are exceedingly important in mitigating the specific congestion condition. Many but not all the studies had hot spots to some degree. In some cases, hot spot locations correspond to a single bus. The existence of hot spots is not dependent on the location of available generators. It is a property of the network itself. By carefully dealing with the existing hot spot and cold spot, the proposed method results in very nice zonal separation. The buses in the original SP15 zone can be distinctly separated to two zones.

By using 5% rule, CAISO has separated the original zone SP15 to two zones, ZP26 and new SP15, which is shown in Figure 3.1. Compare the results based on the proposed method and the new ZP26 and new SP15 zones, the zonal structures are same: the low price zone is the same as new zone ZP26, high price zone plus the "hot spots" and "cold spots" are the same as new zone SP15. This example demonstrates the efficiency of the proposed method in creating zones.

7.4 Illustration of the Methodology on the California System

For the California system, there are a number of identified constraining paths. In addition, the system limits are expressed as "nomograms" that determine feasible operating regions. The nomograms are merely graphic expressions of some limit that has been "mapped" into a smaller set of constraints on readily measurable variables such as voltages, flows, sums of flows, levels of nodal generation or some such measurable quantity. For the purpose of this thesis, we will assume that all nomograms can be expressed as a finite set of limits on linear functions of flows. More specifically, the limit for every path component is illustrated; the case of a limit on the total path power is also studied. However, it is straightforward to extend the method to consider nonlinear functions of these flows, alone or in combination. It is also possible to include power injection or extraction at any node, to add reactive powers and voltage magnitudes, or any other desired system quantity of importance from any nomogram. The fundamentals of the method for zone determination are not affected.

For the cases studied, all paths in the California system have been decomposed into their constituting components. In addition to considering every line in every inter-zonal path as a possible congesting facility, we also consider the possible inter-zonal congestion of what has congested where that has required either invoking RMR, OOS or OOM re-dispatch.

Most of the flowgates of the system are analyzed. The individually congested lines have been selected from among those that form part of cutsets (paths) as defined by the CAISO.

For this study, the following examples considered:

- A simple case of intra-zonal congestion that results in well-defined zones.

- A case of intra-zonal congestion that results in "hot spots," that is, locations in the system with drastically different prices than other surrounding locations.
- A case of inter-zonal congestion.

Only a few cases are shown here, however, similar results for over 100 cases are obtained. The cases that follow are divided into "path" or "nomogram" congestion cases, and "intra-zonal" cases. For each of the "path" cases, we illustrate the following:

- The case of every path or nomogram *component* congesting independently.
- The case of the sum of flows congestion in all components congesting.

For the case of intra-zonal congestion, only the patterns resulting from the individual component congestion are illustrated, since these congested lines are not part of a nomogram or existing path.

7.4.1 Examples

Example 1:

Figure 7.7 to Figure 7.10 show the example of flowgate from bus TESLA to bus TESLA D. Figure 7.7 shows the picture of the exact nodal prices when the particular line congests and a particular generator pair is selected for re-dispatch. Important: the main results are NOT sensitive to the choice of re-dispatch generation pair. To illustrate this point, we also show the nodal price pattern that develops if a less effective generation pair is randomly chosen. This clearly demonstrates that, although the individual prices vary according to this choice, the price pattern that develops does not. This is a direct consequence of the Chao and Peck results [47], where nodal prices are seen as nothing more than a mapping from the shadow price across the congested facility as they get mapped to the individual nodes.

Figure 7.8 illustrates the sorted pattern or price differences across the 50 most significant lines (how many lines or transformers show large price differentials). The biggest price differential should always be across the congested facility (or across a line directly in parallel with the congested facility)

Figure 7.9 is a sorted plot of all nodal spot prices classified by color according to the zones selected by our zone-partitioning algorithm.

Figure 7.10 is a map of California illustrating (using colors) the various zones implied by the zone-partitioning algorithm.

The vertical scales are not significant except in relative terms. However, when values of price differences are much greater than 1 this is an indication that the available generators had difficulty coping with the congestion by means of generation re-dispatch. In cases where there is a single effective generator location, this can be interpreted as a degree of local market power.

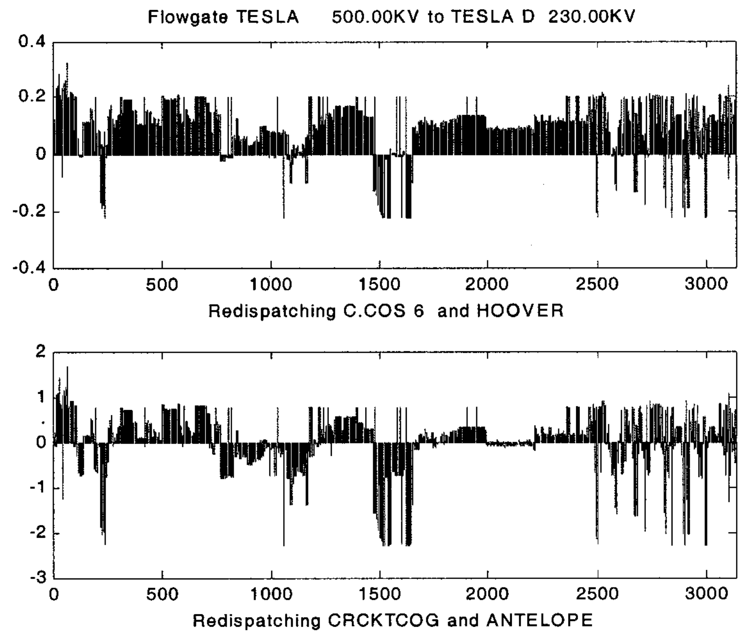

Figure 7.7: Nodal prices

(X axis: Bus Number; Y axis: Nodal Price ($/MW))

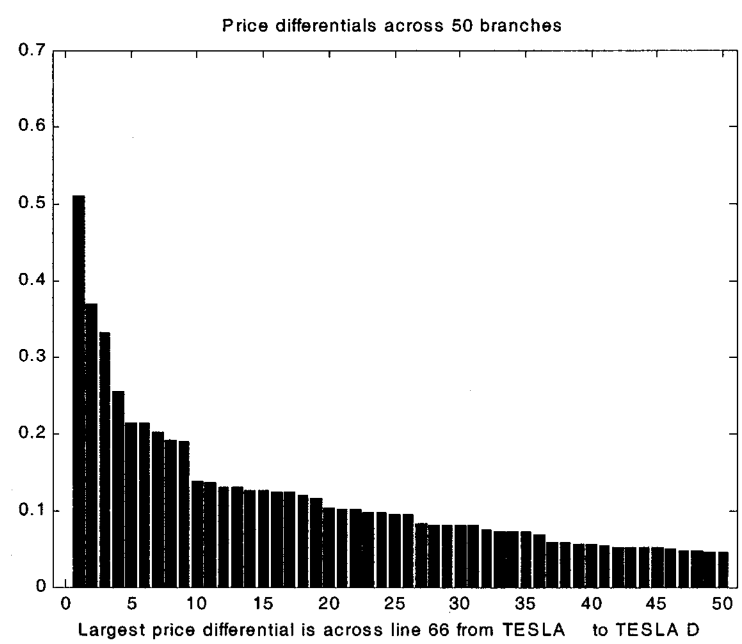

Figure 7.8: Price differential across 50 most significant lines

(X axis: Sorted Line Number; Y axis: Price Differential across Line ($/MW))

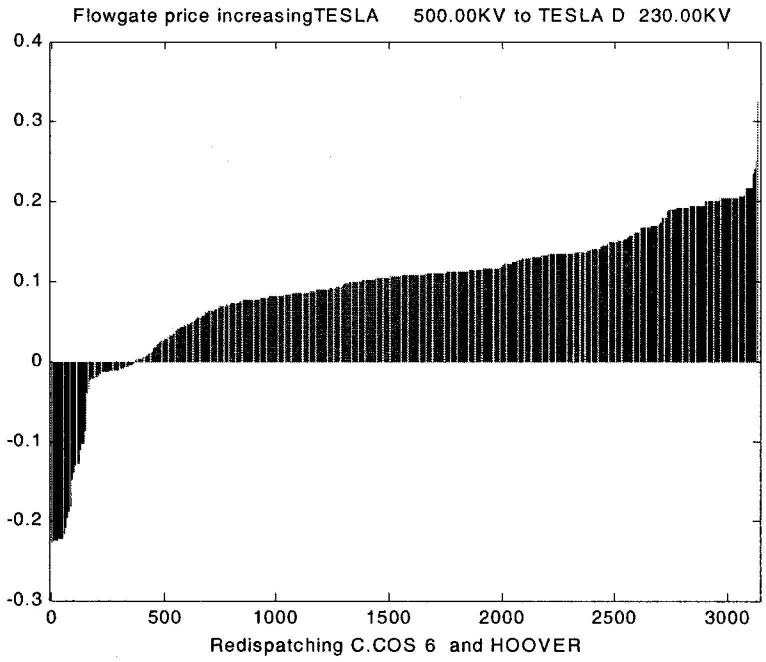

Figure 7.9: Sorted nodal prices organized into zones

(X axis: Sorted Bus Number; Y axis: Nodal Price ($/MW))

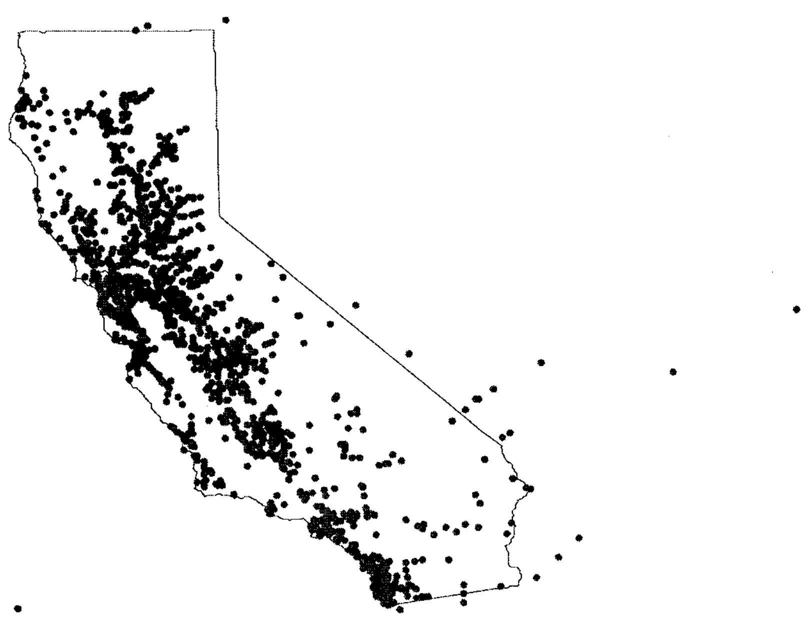

Figure 7.10: Zonal structure that results from congestion.

The system has been organized into 6 zones.

Example 2:

Figure 7.11 to Figure 7.14 show the similar results on the flowgate from bus VINCENT&5 to bus MIDWAY, which belongs to Path 26. Section 7.3 showed how this flowgate determines the zonal separation of the old SP15 zone, the effect on the whole California system is demonstrated here.

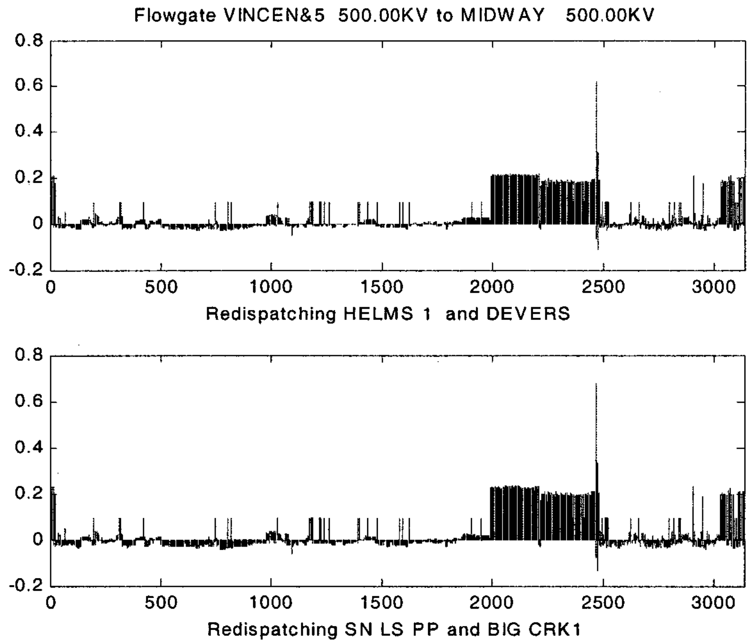

Figure 7.11: Nodal prices

(X axis: Bus Number; Y axis: Nodal Price ($/MW))

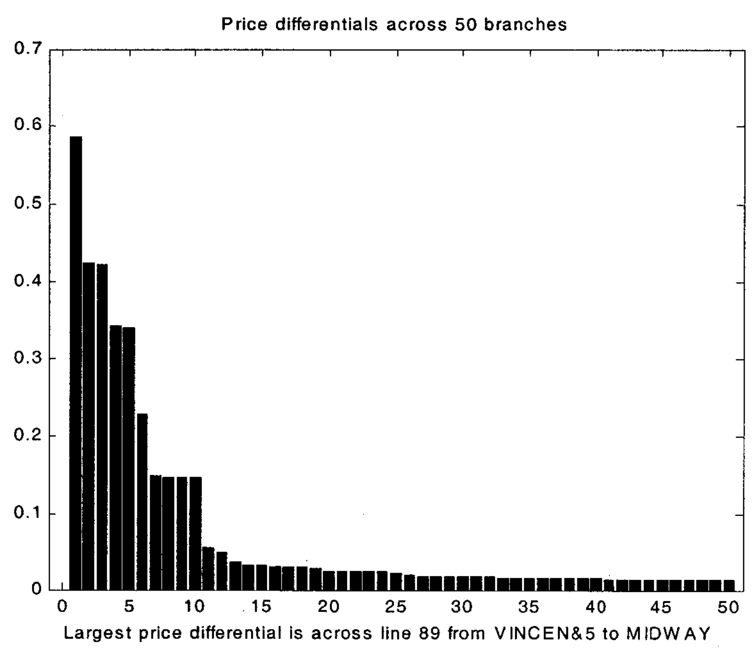

Figure 7.12: Price differential across 50 most significant lines

(X axis: Sorted Line Number; Y axis: Price Differential across Line ($/MW))

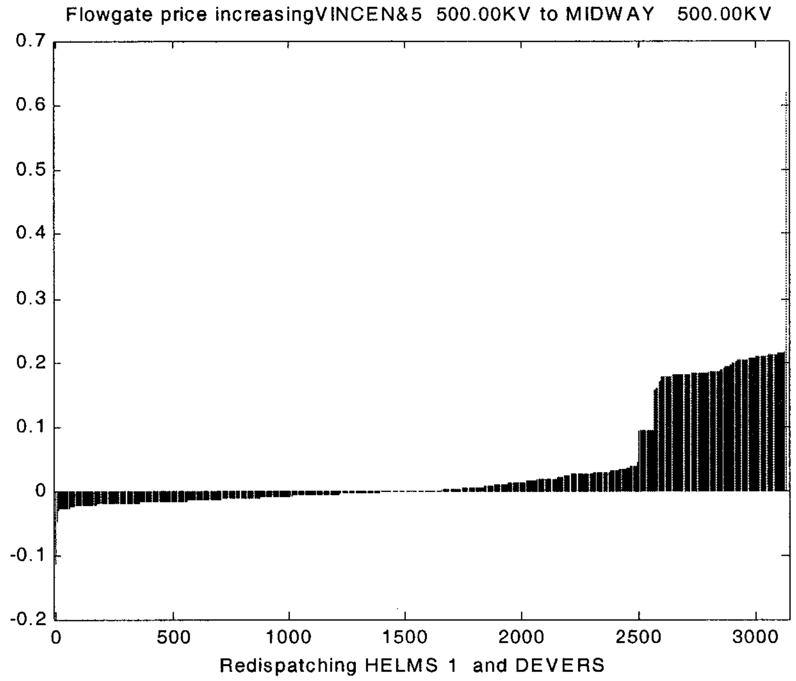

Figure 7.13: Sorted nodal prices organized into zones

(X axis: Sorted Bus Number; Y axis: Nodal Price ($/MW))

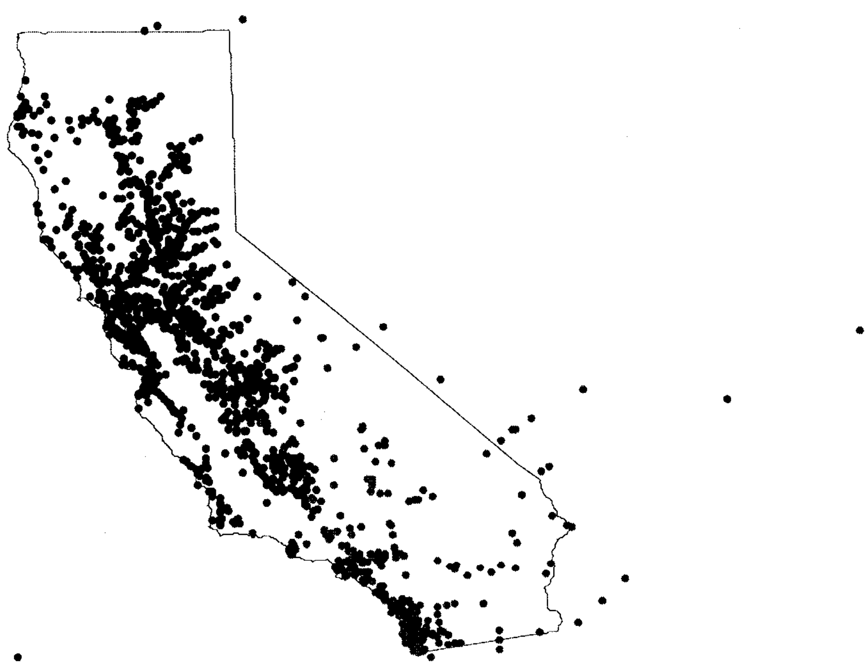

Figure 7.14: Zonal structure that results from congestion.

The system has been organized into 4 zones.

7.4.2 Comparison with the CAISO Zones

Figure 3.1 shows the CAISO zone map, effective from February 2000. This section compares the results shown in Figure 7.7 to Figure 7.10 with the CAISO zonal separations. In that example, based on flowgate from bus TESLA to bus TESLA D, the system is separated into 6 zones. The bus numbers of the zones are 86, 63, 188, 2250, 545 and 2. NP15 zone in CAISO as shown in Figure 3.1 is used as an example to demonstrate the inefficiencies of separating zones based on geographic location.

A major goal of zonal separation is that the nodal prices of the buses in one zone to be as close to a uniform value as possible. A statistical variable measuring how far the data is from one uniform value across the set is the standard deviation. Standard deviation of a data vector **X** is defined as (7.1), n is the number of elements in vector **X**.

$$S = \left(\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{\frac{1}{2}} \tag{7.1}$$

The standard deviation of the nodal prices of all the buses in the NP15 zone is calculated. The standard deviation of buses in all the zones is Figure 7.10 is also calculated. Table 7.1 shows the standard deviation of different zones. From the table, it is clear that the standard deviation in NP15 zone is much higher than the zones generated by the proposed zone-partitioning algorithm, it's even higher than the whole California system.

Table 7.1: Standard Deviation of Nodal Prices

Group #	Standard Deviation
Zone 1	0.0151
Zone 2	0.0185
Zone 3	0.0138
Zone 4	0.0374
Zone 5	0.0181
Zone 6	0.0264
Entire California System	**0.0843**
NP15 Zone	**0.1174**

This example clearly demonstrates that the establishment of a zonal structure should be based not on geography but on the detailed analysis of nodal price patterns, which depend on the generation and the electrical characteristics of the network itself.

7.4.3 Connectivity of the Zones

This section investigates the connectivity of the zones generated by the proposed method. It is necessary to review some background materials in graph theory first [54].

7.4.3.1 Review of Graph Theory

Consider an undirected, unweighted graph $G = (N, E, W_N, W_E)$ without self edges (i,i) or multiple edges from one node to another. N is the set of nodes, E is the set of edges (i,j) connecting nodes, W_N are the node weights, a nonnegative weight for each node, and W_E are the edge weights, a nonnegative weight for each edge. Power system network can be considered as a graph, each bus is a node, and each transmission line is an edge. The following two matrices related to this graph are defined:

Definition 1: The *incidence matrix* In(G) of G is an |N|-by-|E| matrix, with one row for each node and one column for each edge. Suppose edge e = (i,j). Then column e of In(G) is zero except for the i-th and j-th entries, which are +1 and –1, respectively.

Note that since G is undirected, writing edge (i,j) instead of (j,i) is equivalent to multiply column e of In(G) by –1, This ambiguity will not be important to the work in this thesis.

Definition 2: The *Laplacian matrix* L(G) of G is an |N|-by-|N| symmetric matrix, with one row and column for each node. It is defined as follows:

- L(G) (i,j) = degree of node i, if $i = j$ (number of incident edges)
- L(G) (i,j) = -1, if $i \neq j$ and there is an edge (i,j)
- L(G) (i,j) = 0, otherwise.

The following therom state some important facts about In(G) and L(G).

Theorem 1: Given a graph G, its associated matrices In(G) and L(G) have the following properties:

1. L(G) is a symmetric matrix. This means the eigenvalues of L(G) are real, and its eigenvectors are real and orthogonal.
2. Let $e = [1, \ldots, 1]^T$, the column vector of all ones, then L(G) * e = 0.

3. $In(G) * In(G)^T = L(G)$. This is independent of the signs chosen in each column of In(G).

4. The eigenvalues of L(G) are nonnegative: $0 \leq lambda_1 \leq lambda_2 \leq \ldots \leq lambda_n$.

5. The number of connected components of G is equal to the number of $lambda_i$ equal to 0. In particular, $lambda_2 \neq 0$ if and only if G is connected.

6. The number of connected components in graph G is equal to the rank deficiency of In(G) and L(G). The null vectors of L(G) provide information about what nodes are in the various connected components.

Examples:

The following examples demonstrate the above definitions and theorem.

1. Graph G1 has 9 nodes, 12 edges.

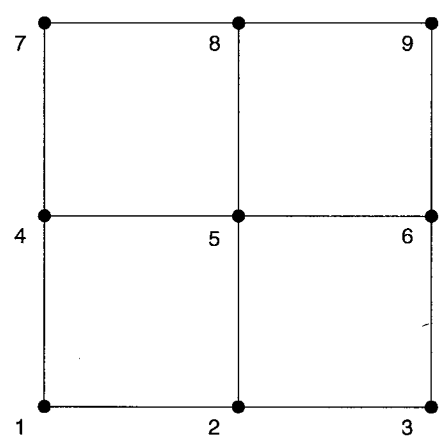

Figure 7.15: Graph G1

Incidence matrix of G1 is:

$$\mathrm{In(G1)} = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \tag{7.2}$$

Laplacian Matrix of G1 is:

$$\mathrm{L(G1)} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 3 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix} \tag{7.3}$$

Rank of L(G1) is 8, the null vectors of L(G1) is:

$$\mathrm{null(L(G1))} = \begin{bmatrix} -0.3333 \\ -0.3333 \\ -0.3333 \\ -0.3333 \\ -0.3333 \\ -0.3333 \\ -0.3333 \\ -0.3333 \\ -0.3333 \end{bmatrix} \tag{7.4}$$

Graph G1 is connected. The size of L(G1) is 9 by 9, and the rank deficiency of L(G1) is 1 (= 9-8). All the elements of the null vector are same, which indicates that all the nodes are in the same zone.

2. Graph G2 has 9 nodes, 7 edges.

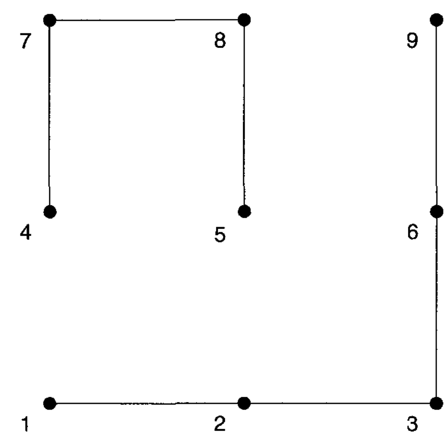

Figure 7.16: Graph G2

Incidence matrix of G2 is:

$$\mathrm{In(G2)} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \tag{7.5}$$

Laplacian Matrix of G2 is:

$$L(G2)=\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{7.6}$$

Rank of L(G2) is 7, the null vectors of L(G2) is:

$$\mathrm{null}(L(G2))=\begin{bmatrix} 0.4821 & -0.1295 \\ 0.4821 & -0.1295 \\ 0.4821 & -0.1295 \\ -0.1447 & -0.4786 \\ -0.1447 & -0.4786 \\ 0.4821 & -0.1295 \\ -0.1447 & -0.4786 \\ -0.1447 & -0.4786 \\ 0.4821 & -0.1295 \end{bmatrix} \tag{7.7}$$

G2 is partitioned to two zones, one zone includes nodes 1, 2, 3, 6, 9, and the other zone includes nodes 4, 5, 7, 8. The rank deficiency of L(G2) is 2. In the first null vector, elements No. 1, 2, 3, 6, 9 all equal to 0.4821, elements No. 4, 5, 7, 8 equal to –0.1447, which reflects the partition of graph G2.

3. Graph G3 has 9 nodes, 6 edges.

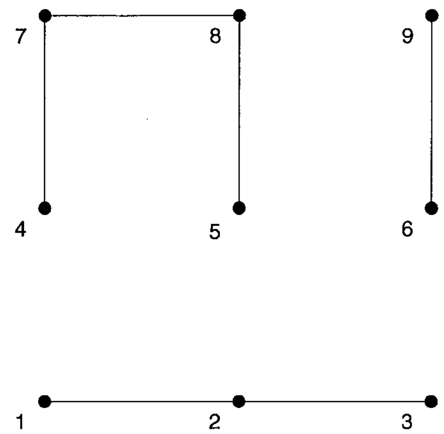

Figure 7.17: Graph G3

Incidence matrix of G3 is:

$$\mathrm{In}(G3) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \tag{7.8}$$

Laplacian Matrix of G3 is:

$$L(G3) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{7.9}$$

Rank of L(G3) is 6, the null vectors of L(G3) is:

$$null(L(G3)) = \begin{bmatrix} 0 & -0.5774 & 0 \\ 0 & -0.5774 & 0 \\ 0 & -0.5774 & 0 \\ -0.4560 & 0 & 0.2051 \\ -0.4560 & 0 & 0.2051 \\ 0.2901 & 0 & 0.6449 \\ -0.4560 & 0 & 0.2051 \\ -0.4560 & 0 & 0.2051 \\ 0.2901 & 0 & 0.6449 \end{bmatrix} \tag{7.10}$$

G3 is partitioned to three zones, zone 1 includes nodes 1, 2, 3, zone 2 includes nodes 6, 9, zone 3 includes nodes 4, 5, 7, 8. The rank deficiency of L(G3) is 3. In the first null vector, elements No. 1, 2, 3 equal to 0, elements No. 4, 5, 7, 8 equal to –0.4560, elements No. 6, 9 equal to –0.2901, which reflects the partition of graph G3.

7.4.3.2 Connectivity of the California Zones

The California system studied in this thesis has 3134 buses and 3995 branches. The size of the Laplacian matrix is 3134*3134; rank of the Laplacian matrix is 3116, which means the system is separated to 18 (= 3134-3116) zones. Further analysis shows that there are 17 single bus islands, and the rest of the system is connected, as shown in Figure 7.18.

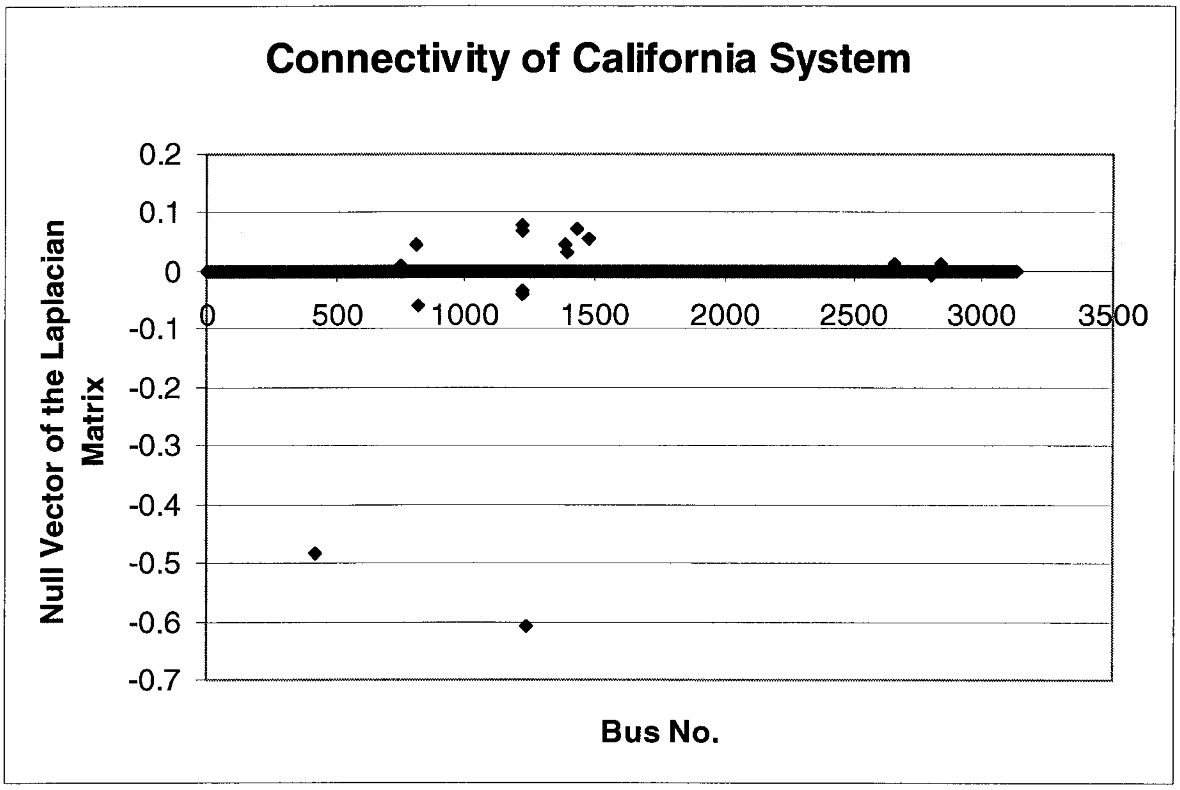

Figure 7.18: Connectivity of California system

Consider Example 1 of Section 7.4.1, shown in Figure 7.7 to Figure 7.10, which is based on the flowgate from bus TESLA to bus TESLA D. The system is separated into 6 zones, and there are 86, 63, 188, 2250, 545 and 2 buses in each zone. After cutting all the transmission lines between different zones, the rank of Laplacian matrix of each zone is shown in Table 7.2. One can see that Zone 1 and Zone 6 are connected, Zone 2 to Zone 5 are not directly connected.

Table 7.2: Rank of Laplacian Matrix of Zones – Flowgate TESLA to TESLA D

Zone	Buses	Rank of Laplacian Matrix	Connectivity
1	86	85	Connected
2	63	57	Not Connected
3	188	173	Not Connected
4	2250	2239	Not Connected
5	545	492	Not Connected
6	2	1	Connected

Similar for Example 2 of Section 7.4.1, which is based on flowgate from bus VINCENT&5 to bus MIDWAY, the system is separated into 4 zones, and there are 2568, 562, 3, and 1 buses in each zone. After cutting all the transmission lines between different zones, the rank of Laplacian matrix of each zone is shown in Table 7.3. One can see that Zone 2 is connected, Zone 4 is a single bus, and Zone 1 and Zone 3 are not directly connected.

Table 7.3: Rank of Laplacian Matrix of Zones – Flowgate VINCENT&5 to MIDWAY

Zone	Buses	Rank of Laplacian Matrix	Connectivity
1	2568	2538	Not Connected
2	562	561	Connected
3	3	0	Not Connected
4	1	0	Single Bus

The results in Table 7.2 and Table 7.3 show that after all the transmission lines between different zones are cut, some buses in one zone are not connected. What this really means is that some buses in one zone are not connected directly. But as shown in Figure 7.18, the system is connected as a whole except some islanded single buses. So even some buses in one zone are not connected directly, they are connected indirectly through buses in other zones anyway.

7.4.4 Comparison of Nodal and Zonal Prices

In this section, the total cost at a true optimum, which yields different nodal prices at every node, is compared against the sub-optimal objective function value achieved when uniformed prices are applied in each zone, as determined by the partitioning scheme in this thesis.

A zonal price, can be defined according to several different approaches:

a. Determining the non-constrained price, which is the price with all congestion ignored, followed by an uplift calculation. This is the method adopted in [13, 14]. Uplift is the constrained on and off payment of the generators assigned to increase and reduce their generation, because of the network constraints.

b. Average price. An average of all nodal prices leads to the zonal price. It is important to mention that all the prices are taken into consideration, regardless the capacity of each generating unit.

c. Weight approaches. Two methods may be proposed.

c1. The zonal price is given by:

$$\frac{\sum_{1}^{n}(P\max * NP)}{\sum_{1}^{n} P\max} \tag{7.11}$$

where Pmax stands for the maximum generation capacity of each generator unit and NP is the nodal price ($/MW) of the same unit, n is the number of units considered.

c2. In this case, the maximum capacity of each generator unit is replaced by the actual power generated at each unit (Pgen), yielding:

$$\frac{\sum_{1}^{n}(Pgen * NP)}{\sum_{1}^{n} Pgen} \tag{7.12}$$

Methods (a) [13] [14] and (c2) [9] discussed above are widely used in the power markets. In this thesis, method (c2) is used to calculate zonal prices.

Consider Example 1 of Section 7.4.1, shown in Figure 7.7 to Figure 7.10, which is based on the flowgate from bus TESLA to bus TESLA D. The system is separated into 6 zones. In Figure 7.7, assume the unit of nodal price is $/MW (The purpose of the discussion in this section is to show the ideas, the absolute values are not important, the comparison between values is what really matters). Table 7.4 summarizes the total cost of all the loads in each zone.

Similar results of Example 2 of Section 7.4.1, which is based on flowgate from bus VINCENT&5 to bus MIDWAY. The system is separated into 4 zones. Table 7.5 summarizes the total cost of all the loads in each zone.

Table 7.4 and Table 7.5 demonstrate that the total cost resulting from zonal prices is higher than that from nodal prices, which means that zonal price does not result in optimal cost in power markets.

Table 7.4: Total Cost by Nodal and Zonal Prices – Flowgate TESLA to TESLA D

	Total Cost ($)	
Zone	**Nodal Price Approach**	**Zonal Price Approach**
1	-0.6520	-0.6312
2	-0.2161	-0.7140
3	-0.4516	-0.2792
4	35.8410	37.4879
5	2.3756	2.8571
6	0	0
Total Cost	**36.8970**	**38.7206**

Table 7.5: Total Cost by Nodal and Zonal Prices – Flowgate VINCENT&5 to MIDWAY

	Total Cost ($)	
Zone	**Nodal Price Approach**	**Zonal Price Approach**
1	0.0997	0.7534
2	40.5031	40.4010
3	0	0
4	0	0
Total Cost	**40.6028**	**41.1544**

Inside one zone, the generators whose bid prices are higher than the zonal prices would like to shut down; on the other hand, the generators whose bid prices are lower than the zonal prices would enjoy extra benefits, they would like to generate as much as possible, may against the generation limit. This also indicates that zonal price is not the optimal approach.

7.5 General Observations and Conclusions

The results from the studies lead to a number of observations and conclusions.

- The location of the re-dispatching generating units has little to do with the resulting zonal pattern that results (although changing the location of the marginal generating units can lead to different price differentials, the patterns stay fundamentally the same).
- In some cases crisp well-defined zone boundaries occur. However, in other cases the boundaries are not that crisp.
- In general, intra-zonal congestion did not result in good zone separation. Intra-zonal congestion when the sum of the power across the path is congested often led to a reasonable zonal structure except for the common presence of frequent "hot spots."
- These hot spots correspond to locations that are exceedingly important in mitigating the specific congestion condition. Many but not all the studies had hot spots to some degree. In some cases, hot spot locations correspond to a single bus.
- The existence of hot spots is not dependent on the location of available generators. It is a property of the network itself.
- Even when an entire path that presumably separates the system, significant price dispersion remains as a result of flows in the external system.
- The best organization of zones is not always according to pure geography, but according to electrical connectivity and topology. In some cases better zones are

created when some high voltage buses that are geographically within one zone are actually placed in a different zone.

- In the studied system, the total cost results from zonal prices is higher than that from nodal prices. This fact confirms that zonal price is not the optimal pricing system in power system markets.

7.6 Comments on Specific Paths and Flowgates

- Congestion on some path 15 components leads to zones that are not well defined and vary widely depending on the component. For some components, such as GATES to HENRETTA, good zonal separation develops. Congestion of the total path flow leads to a better but imperfect separation of the system into two reasonably well-defined zones with some "hot spots."
- Congestion of either line to CFE results in clear separation of markets.
- Paths 26 and 44, when they congest on the total sum of powers condition, lead to reasonably crisp zonal separation of the system.
- Congestion into the Fresno area assessed as the sum of total imports leads to a relatively well-defined zone separation of the system into two zones. However, if some of the limits are surrogates for voltage problems a different and less crisp picture might evolve.
- Intra-zonal congestion in some paths such as Humboldt or some intra-zonal congestion such as GRIZ to Caribou leads to highly localized effects that do not propagate far into the system.

- Other cases of intra-zonal congestion lead to much less crisp potential zonal boundaries.

7.7 Recommendations

Based on these studies, some preliminary recommendations can be issued, regarding the creation and aggregation of zones in general:

1. The establishment of a zonal structure should be based not on geography but on the detailed analysis of nodal price *patterns*, which depend on the generation and the electrical characteristics of the network itself. The use and knowledge of precise nodal prices is not necessary.
2. The price patterns should be established based on prevailing and anticipated congestion patterns as determined by existing or new operational procedures that assure system security. Those procedures deemed at present too complex for direct incorporation into the methodology will require further analysis to "translate" them into meaningful and practical measures in terms of limits on traded quantities or combinations of traded quantities.
3. A zonal structure established according to the terms above should be sufficient for most needs. However, the studies here suggest that the presence of "hot" (or "cold") spots (that is, locations that have a marginal price that is significantly above or below the otherwise zonal value) need to be recognized. Thus, a "zonal plus hot spots" approach to zonal pricing is recommended. If using prices to induce or

restrict operation at these hot spots is rejected, then a means for dealing with these "hot spots" by regulatory means needs to be developed.

4. There should be an effort undertaken to try to go back to basic principles in the establishment of operational limits. Nomograms created for an era of regulated operation when computing and modeling capabilities were limited should be gradually replaced with new nomograms and other ways of expressing system limits based on actual modern system operational practices. These limits should be based on properly agreed upon reliability criteria and should be expressed in a manner sufficiently accurate but also sufficiently simple for the market to take into consideration when trading decisions are made.

Appendix A illustrates some of the main nomograms in use by the CAISO.

Chapter 8

Conclusions and Future Research

8.1 Conclusion

Congestion management of modern power system is a complicated problem. It involves the physical structure of the power system, the financial situation, the strategy of customers, the economic behavior of market participants, government policies, operational rules, and many other factors. Price-driven congestion management is a major congestion management method used in modern power system markets. Nodal and zonal pricing are the two major pricing systems used for this purpose.

The major contributions of this thesis are:

- For those cases where it is decided that a zonal pricing system is to be used, this thesis proposed an Automatic Zone Creation/Merging/Partition Methodology, which is based on the fact that the *nodal price patterns* associated with zonal and nodal prices are largely a function of the network and do not strongly depend on the prices at the various generators. Thus, *it is possible for the most part to separate the* *concept of zone partitioning from the cost and location of individual generators.* The best organization of zones is not always according to pure geography or to political or institutional boundaries, but according to electrical connectivity and topology.

- This thesis developed an OPF model including price caps based on the duality theory of Linear Programming. This model clearly revealed how price caps affect nodal prices, generation, and load. It illustrated not only the price distortions that occur as a result of price caps, but the inability to solve the problem in several cases with price caps unless "extra generation" (however obtained) or "load shedding" is assumed.
- This thesis thoroughly summarized the zonal and nodal price calculation, including theoretical fundamentals, different methods, relations between these methods, and how transmission congestions and losses affect nodal prices. The comparison of these two pricing systems was also performed. All the concepts are demonstrated through examples. This part of work is an excellent reference for readers who want to understand this complicated problem.

Several conclusions can be made based on this work:

- The similarities of Zonal and Nodal pricing system are more significant than the differences. Generally speaking, nodal pricing is more cost efficient than zonal pricing.

- A “fixed rule” for zone creation (such as the 5% rule and market power mitigation methods used by CAISO at the time of the initial writing of this thesis) are (and will always be) in need a lot of continued improvements and adjustments.
- The price cap systems (such as those used in California) must always consider the effect of many potential factors, which can drive the price high, and generate a lot of gaming opportunity for the generators. Any price cap system implementation must consider the serious implications on dispatch and gaming that are provided by any such scheme.
- If losses are ignored, OPF method and the linear approximation method give identical nodal prices with identical constraining elements. When losses are considered, the two methods will give different results unless the effect of losses in explicitly brought into the linear approximation models by means of penalty factors.
- The duality theory in Linear Programming is the tool to model price caps in the power system markets. Implementing price caps on the nodal prices is equivalent to implementing upper bound on a function of Lagrange Multipliers of the primal problem, which are the variables in the dual problem. By solving the dual of the modified dual problem, we can find how the price caps affect the power system.
- The Automatic Zone Creation/Merging/Partition Methodology proposed in this thesis gives a new view of how to create, define and breakup zones and how to perform zonal congestion management.
- The system Jacobian matrix [35] includes the instantaneous information of the system itself, it has Laplacian format [54, 55, 56]. PTDFs are nonlinear, by using the

approximate method (4.3), the transient status of the system is not reflected in PTDF. If the exact PTDF calculation method (4.2) is used, the transient system information can be reflected in the PTDF, but this does not make major difference during the nodal price calculation in most of the cases we examined. Nodal prices are related to the system Jacobian, because they reflect different aspects of the same system. However, the relationship is not direct, the direct factors that determine flowgate shadow prices are flowgate constrains and generator bid prices, which further determine the system nodal price pattern.

8.2 Future Research

There are still many unsolved and new problems in this area, possible extensions to this work include:

- For cases where one wishes to use zones for congestion management, it is essential to improve the proposed methodology of Automatic Zone Creation/Merging/ Partition, make this method more practical to implement and more dynamic and adaptable to changing system realities.
- The impact of flow regulation devices on zone creation and definition is huge and needs to be considered. It would be interesting and important to include the effects of market power in the methodology.
- Implement the results in Chapter 6 in a more realistic system. Besides the hard price caps, also study the effect of the soft price caps, (such as, for example, the "cost + $25" that was used for a time in California).

- The dynamics and stability of power system markets is an active research area [52, 53]. Inappropriate price caps may have significant impact on the market dynamics. Combining previous work in this area with the work in Chapter 6 can help understand and resolve this problem.
- As pointed out in Chapter 1, market power can have significant impact in the electricity markets. It could also affect transmission congestion patterns, change nodal prices, and further change zonal separation boundaries. Modeling and understand how market power affects the nodal and zonal prices is an extremely important and key area of research, but one that was considered outside the scope of this thesis. It deserves, however, a great deal of attention by future researchers.

Appendix A

Nomogram

A.1 Introduction

The nomogram is very like the Safe Operating Area (SOA) plot of the power electronics devices, such as power diode, MOSFET and others. It gives the relationship of the limits of the different transmission lines. These transmission lines may be DC or AC.

For the system operators, the limits should be considered include both nomogram and OTC. If the limit violation occurs, the system must be adjusted to return within the nomogram limits within 10 minutes. The nomogram is affected by many other factors, including percentage of hydro, heavy duty or super heavy duty, etc.

This appendix collected the major nomograms used in the CAISO. All the information is from CAISO web site [9].

A.2 AC/DC Nomogram

This refers to the COI/NW-Sierra and PDCI north-to-south operating Nomogram. The purpose of this nomogram is to operate in conformance with the WSCC Minimum Operating Reliability Criteria (MORC) and to protect the WSCC system during heavy export conditions from the Northwest to the Southwest from disturbances similar to those that occurred on July 2 and August 10, 1996. The nomograms are based on the loss of the Bipolar PDCI, a 2-unit loss at Palo Verde, a 2-unit loss of Moss Landing, or a Table Mountain South Double Line Outage. The Nomograms are limited by post-transient Outages.

The following websites are the nomograms at different situations.

1. 2001 heavy summer AC/DC monogram (As a function of the percentage of Northern California Hydro Generation, for Northern California area load less than 21,250 MW):

http://www.caiso.com/docs/2000/12/21/2000122113332111161.pdf

2. COI/PDCI North of John Day Nomogram:

http://www.caiso.com/docs/1999/09/25/1999092510485911117.pdf

3. AC/DC Nomogram:

http://www.caiso.com/docs/1999/09/25/1999092510581811842.pdf

A.3 West of Borah Verse Path 15 Nomogram

The purpose of the West of Borah/Path 15 Nomograms is to operate in conformance with the WSCC Minimum Operating Reliability Criteria (MORC). The Nomograms are based on the Los Banos South 500 kV Double Line Outage (Los Banos – Midway / Los Banos – Gates 500 kV lines), the Malin South 500 kV Double Line Outage (Malin- Round Mountain 500 kV lines), the Los Banos North 500 kV Double Line Outage (Los Banos – Telsa / Los Banos –Tracy 500 kV lines), or a Bipolar PDCI Outage. The Nomograms are limited by the reactive margin in PG&E's southern transmission system and IPC's transmission system following post-transient Outages, and the thermal limits on PG&E's Gates – Panoche 230 kV lines and PG&E's Gates 500/230 kV transformer bank.

http://www.caiso.com/docs/2000/12/21/2000122113345811315.pdf

A.4 East-of-River / Southern California Import Transmission Nomogram (SCIT)

This Operation Procedure describes limits on Southern California imports based on the parameters of the East-of-River / Southern California Import Transmission Nomogram (SCIT) for the 2001 summer season (Effective date TBD).

The East-of-River / Southern California Import Transmission Nomogram (SCIT) Nomogram became effective on October 1, 1991, replacing the retired West Of the River (WOR) Nomogram.

This Nomogram monitors the following:

- Power flow on five major paths into Southern California area.
- System inertia in Southern California area.
- Actual East Of the River (EOR) flows.

1. East-of-River / Southern California Import Transmission Nomogram (SCIT) (1)

   http://www.caiso.com/docs/1999/09/25/199909251050441424.pdf

2. East-of-River / Southern California Import Transmission Nomogram (SCIT) (2)

   http://www.caiso.com/docs/2000/12/21/200012211328339829.pdf

3. SCIT Nomogram Axis and Reduction Tables:

   http://www.caiso.com/docs/2000/10/05/2000100513131727510.pdf

A.5 San Diego Area

The purpose of this Operating Procedure and the associated SDG&E / CFE Nomogram is to protect against the thermal overload of the Imperial Irrigation District (IID) 230/161 kV transformer bank for loss of the North Gila – Imperial Valley 500 kV line, and to protect against voltage collapse in the SDG&E Area for loss of G-2 SONGS or loss of Imperial Valley – Miguel 500 kV line.

http://www.caiso.com/docs/2000/10/05/2000100513131727510.pdf

A.6 WSCC Path 45 ISO-CFE Operating Transfer Capability and Nomogram

Study results show that P-45 is limited to 408 MW bi-directional and is a 30-minute thermal limit. The 408 MW OTC is a subset of the total SDGE/CFE OTC described in San Diego Area procedure T – 132. CAISO will notify CFE and SDG&E of Schedule limitations and curtailments, and will operate within the parameters of the ISO / CE Operating Nomogram.

http://www.caiso.com/docs/1999/09/25/1999092511103422245.pdf

A.7 Some Abbreviations Used in This Appendix

C3PO:	California Coordinating Committee for Power Operations,
COI/NW-Sierra:	California-Oregon Intertie/Northwest-Sierra,
EOR:	East of River,
IID:	Imperial Irrigation District
MORC:	Minimum Operating Reliability Criteria,
OSS:	Operating Studies Subcommittee,
OTCPG:	Operating Transfer Capability Policy Group,
OTC:	Operating Transfer Capabilities,
PDCI:	Pacific DC Intertie,
SCIT:	Southern California Import Transmission Nomogram,
WOR:	West of River.

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